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# Compressible Flow Measurements Using Nano-scale Thermal Anemometry Probes

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## Abstract

A nano-scale thermal anemometry probe (NSTAP) was designed and fabricated to collect velocity measurements in compressible flows. The NSTAP was tested in the Trisonic Wind Tunnel Munich at high-subsonic and supersonic speeds. The subsonic tests were aimed at the characterization of free-stream turbulence levels in the facility, while the supersonic tests were aimed at investigating the nature of convective heat transfer from the probes. For the range of mass fluxes tested, a linear relationship between the Nusselt number and the Reynolds number was found, contrary to the power law commonly used for describing the heat transfer of conventional hot-wire probes in supersonic flows. This linear relationship has previously been noticed at length scales close to the molecular mean free path and has been attributed to the free-molecule flow range (high Knudsen number).

## 1 Introduction

Ubiquitous in nature, turbulent flows are defined by their chaotic motion, three-dimensionality and diffusive tendencies. Due to the presence of a large range of length and time scales in the flow, turbulent flow measurements are extremely challenging to acquire. In order to extract velocity data at a high frequency, hot-wire measurements are commonly performed. Hot-wires consist of a metallic filament which is heated by the Joule effect. Heat diffuses away due to flow convection and the change in heat transfer registers either as a change in resistance if operated in constant current mode, or as a change in current if operated in constant temperature mode (Comte-Bellot, 1976). Hot-wires have the capability of measuring one velocity component (or two with the use of cross-wires) in flows with a high uni-directional mean velocity. However the use of hot-wires in flows with strong three-dimensional motions can lead to biased results. Particle Image Velocimetry (PIV) is an alternative method of measuring flow velocities which, in contrast to hot-wires, can acquire planar or even volumetric velocity measurements. This allows for multiple velocity components to be measured simultaneously. Furthermore, PIV can be used in separated flows as this technique can determine the velocity direction (Raffel et al., 2018). However, due to temporal resolution limitations associated with PIV and conventional hot-wires, nano-scale hot-wires are a promising technique for obtaining fully-resolved measurements.

With the additional constraints of short time scales and high speeds present in supersonic flows, measurements become more challenging. In order to accurately extract the flow statistics, it is essential to have sufficient sensor response to capture turbulent fluctuations over all frequencies and length scales. With the advancement of nano-technology and semiconductor manufacturing, researchers at Princeton University developed the nano-scale thermal anemometry probe (NSTAP). This sensor has been used successfully in a wide array of incompressible flows and has been shown to have a frequency response greater than 150 kHz, more than an order of magnitude faster than conventional hot-wires (Bailey et al., 2010). Due to its nano-scale thickness and its micro-scale width and length, the NSTAP can also capture the very small length scales present in turbulent flows, minimizing the spatial attenuation effects.

While the NSTAP is a well-established anemometry probe for the analysis of incompressible flows, its applicability to compressible flows has not been previously explored. In compressible flows, the density $\rho$ and the streamwise velocity $u$ can no longer be decoupled using hot-wire anemometry, which adds a level of complexity to the problem. It can be shown from dimensional analysis that the Nusselt number, $Nu$, is a function of the Mach number, $M$, the Reynolds number, $Re$, the Prandtl number, $Pr$, the overheat ratio, $\tau$, and the length-to-diameter ratio, $l/d$:

$$Nu = Nu(M, Re, Pr, \tau, l/d) \tag{1}$$

This is the most general form of the heat transfer equation (Kovasznay, 1950). If $Pr$, $M$ and $l/d$ are constant, $Nu$ becomes dependent only on $Re$ and $\tau$, and the heat transfer relationship reduces to Equation 2:

$$E^2 = L(\tau) + M(\tau)\,[\rho u]^n \tag{2}$$

It must be noted that any polynomial curve can be used to fit the data at a certain overheat ratio, defined below in Equation 3. A power law is most often used due to its physical significance in the incompressible flow community (resembling King's law).

$$\tau = \frac{T_\mathrm{w} - T_\mathrm{e}}{T_\mathrm{o}} \tag{3}$$

where $T_\mathrm{w}$ represents the temperature of the heated wire, $T_\mathrm{e}$ is the temperature of the unheated wire while in flow and $T_\mathrm{o}$ represents the stagnation temperature (Kovasznay, 1950; Smits et al., 1983). Equations 2 and 3 illustrate the dependence of the voltage, $E$, on both the mass flux, $\rho u$, and the total temperature, $T_\mathrm{o}$.

In order to validate the NSTAP for compressible flow analysis, the sensor is tested in both high subsonic and supersonic flow regimes, with the intention of determining the free-stream turbulence level. The free-stream turbulence level of a wind tunnel is an important metric of the flow quality. It is an input to numerical simulations and is used as a comparison tool for other experimental setups. The values of free-stream turbulence are typically low and thereby challenging to measure. A high signal-to-noise ratio is required to be able to distinguish the flow from the background noise. The objectives of this paper are twofold; first, the free-stream turbulence levels are computed using both an NSTAP and PIV for a wide range of Mach numbers $(0.3 < M < 2.0)$, the results are then compared at $M = 0.3$ in order to find a correlation between the mean mass flux and the turbulence intensity. Second, the heat transfer relationship governing the NSTAP is investigated at supersonic speeds. To accomplish the latter, the overheat ratio, $\tau$, and the stagnation pressure, $p_\mathrm{o}$, are varied while keeping the Mach number constant at $M = 2.0$, allowing for direct comparison between tests. Section 2 describes the design changes made to the conventional NSTAP in order to withstand supersonic speeds and describes the experimental facility used in this research. Section 3 shows the results from the subsonic NSTAP/PIV comparison and looks into the heat transfer relationship governing the NSTAP at supersonic speeds.

## 2 Experimental Methods

The NSTAP has a unique capability of measuring streamwise velocity fluctuations at high frequencies $(f > 10^5$ Hz), making it an ideal anemometry probe for high speed flows. The NSTAP is a silicon-based MEMS device made in the Micro/Nano Fabrication Laboratory at Princeton University. Metal sputtering and deep reactive ion etching are performed on a silicon wafer as part of the process to obtain the final sensor seen in Figure 1. Here, platinum is sputtered due to its advantageous electrical and physical properties, such as having a constant temperature coefficient of resistance for a wide range of temperatures and enabling convenient etching of the surrounding silicon structure. At the most upstream location of the sensor lies a metallic filament which is heated above the flow temperature by the Joule effect. The sensing element used in subsonic flows is 2 $\mu$m wide and 100 nm thick, with a length of either 30 or 60 $\mu$m depending on flow characteristics. However, these dimensions can easily be altered, which is done for this study. As it can be seen in Figure 1, the wire is free-standing and held in place by two electrically conductive pads. During operation, the flow convects heat away from the wire and the change in instantaneous flow velocity is read as a change in output voltage. Although the most relevant details are summarized here, please refer to Vallikivi and Smits (2014) or Fan et al. (2015) for information regarding all fabrication processes of the NSTAP.

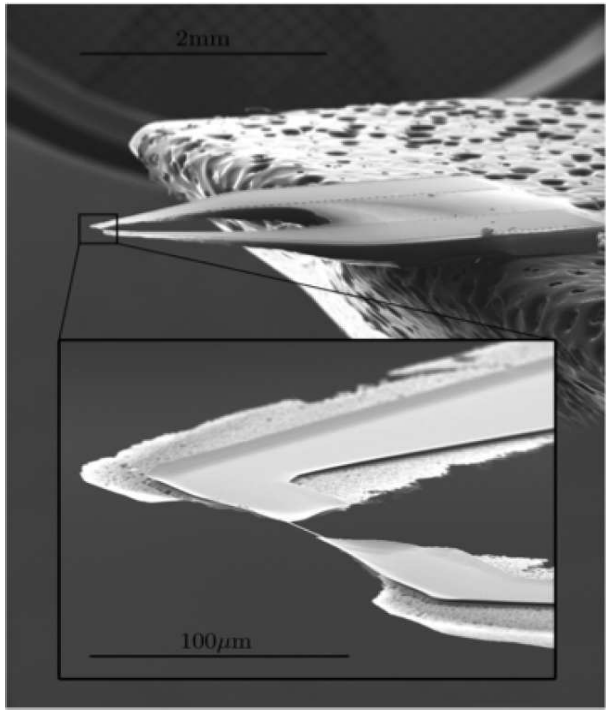



Figure 1: Scanning electron microscope (SEM) image of the NSTAP on carbon tape. The inset image displays the free-standing sensing element supported on each end by metal sputtered onto silicon pads. Figure excerpted from Fan et al. (2015).

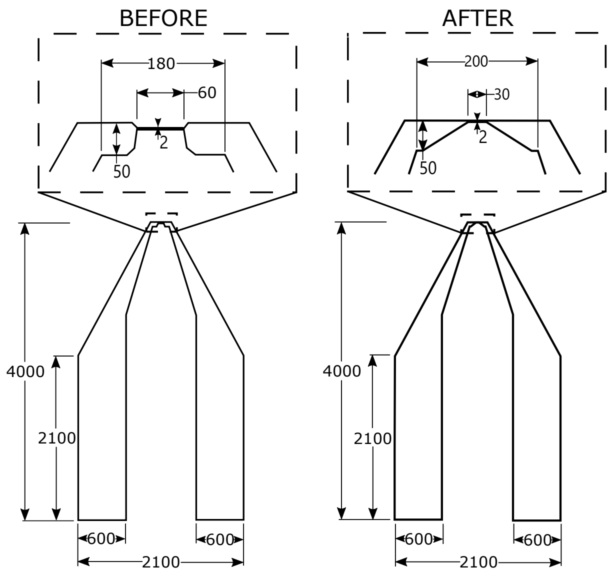
The NSTAP has been tested in a variety of high Reynolds number incompressible flows and has yielded promising data (see for instance Hultmark et al., 2012). As mentioned, the NSTAP has shown to have a significantly higher frequency response and a superior spatial resolution compared to conventional hot-wires due to its small sensing element, support structure and corresponding thermal inertia (Bailey et al., 2010). This property is central to turbulence statistics in supersonic flows, where the time scales are extremely short. Hence, the NSTAP was selected as a suitable sensor to extract well-resolved turbulence statistics in supersonic flows. However, due to severe loading conditions and significant vibrations of the sensor, design iterations were needed to increase the structural rigidity of the sensing element.

## 2.1 Sensor Design Variants

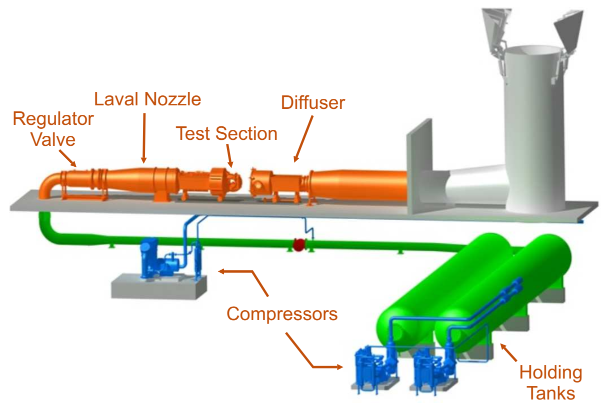
Four modifications were made to the original NSTAP design to prohibit structural failure and improve the flow field characteristics in supersonic flows. First, the thickness of the metallic layer was increased from 100 nm to 500 nm. In order to do this, a new spin coating recipe consisting of a bilayer of LOR10A lift-off resist and AZ1505 photoresist was developed to ensure proper metal lift-off.

Second, due to the shock structure on the sensor assembly, the two-dimensional geometry of the metallic layer was altered (see Figure 2(a)). Since shock waves form at the tip of the sensor when testing at supersonic speeds, the stub design was modified for aerodynamic purposes. The wire was also placed at the most upstream location in order for it to sense unobstructed flow from the shocks which form on the silicon supports.

The final step of fabrication consists of removing the 500 nm thick layer of silicon dioxide ($SiO_2$) from the sensor in order to fully expose the sensing element to flow. This is normally done with the use of a buffered oxide etchant (BOE). After having removed the layer of $SiO_2$ present on one side of the wire, the sensor is fully *etched* - that is, the sensing element is exposed to flow on both sides. Third, unetched sensors − where the layer of $SiO_2$ remains on one side of the wire − were also tested for comparison, as the structural rigidity is significantly improved. However, the frequency response of the unetched sensors decreased, since the sensing element is now only exposed to flow on one side and a portion of the heat is transferred to




(a) 2D Layout of Metal Deposited on Silicon Substrate




(b) Trisonic Wind Tunnel Munich

Figure 2: a) Figure excerpted and adapted from Fan et al. (2015). The schematic on the left depicts the two-dimensional metallic pattern of the NSTAP used in incompressible flows. The schematic on the right is the modified pattern of the NSTAP to withstand supersonic flow conditions. All units are in microns. b) Schematic of the Trisonic Wind Tunnel Munich located at Bundeswehr University Munich. The controllable geometry allows for active changes in Mach numbers, while the regulator valve controls the stagnation pressure and hence, the Reynolds number.

the silicon dioxide layer.

Lastly, the length of wire was set to 30 $\mu$m in order to reduce wire deflection, which can lead to failure. This can also be seen in Figure 2(a). All four variants of the NSTAP were fabricated and tested in the test facility presented in the following subsection.

## 2.2 Trisonic Wind Tunnel Munich

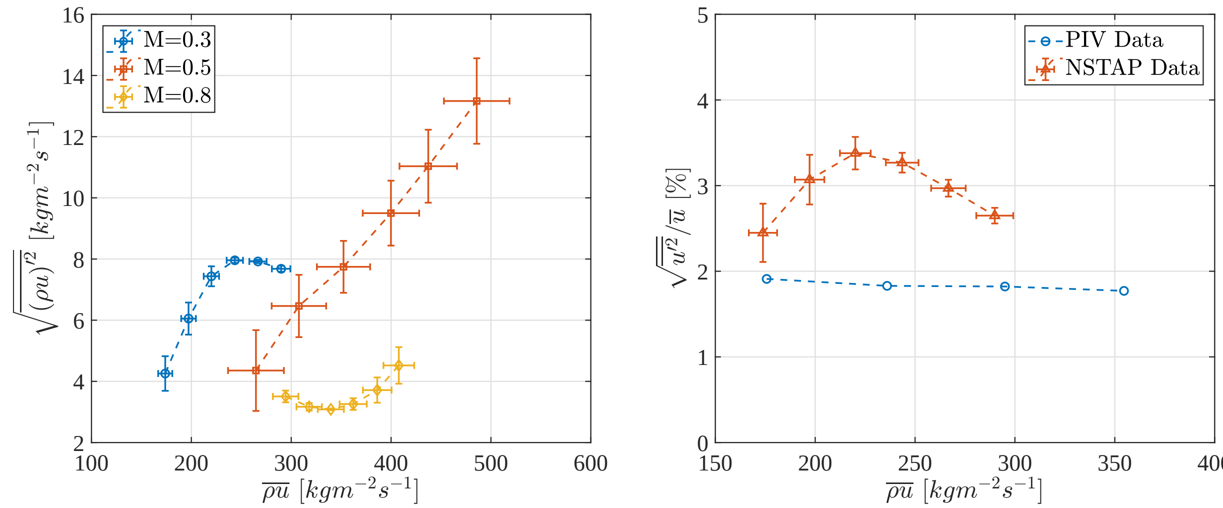
All tests were performed in the Trisonic Wind Tunnel Munich (TWM) located at Bundeswehr University in Munich, Germany. The TWM is a two-throat blowdown wind tunnel which can run in subsonic, transonic and supersonic regimes ($0.15 < M < 3.0$). Air is fed into the tunnel by two large pressurized tanks ($p_o$ up to 20 bar) each containing 178 $m^3$ of air, illustrated in Figure 2(b). The TWM's test section is 300 *mm* wide, 675 mm high and 1700 mm long. An adaptive de Laval nozzle (upstream of the test section) and an adaptive diffuser (downstream of the test section) allow for varying the Mach number during operation of the wind tunnel. The total pressure, ranging from $p_o = 1.2$ to 5.0 bar (which results in a maximum Reynolds number of $80 \times 10^7$ $m^{-1}$), can be varied during wind tunnel operation. The unique capability of varying Mach number and Reynolds number independently and in real-time is beneficial in understanding the effect of Mach number on the heat transfer relationship governing the NSTAP. It also allows for smooth start-up and shutdown of the wind tunnel, which otherwise can damage the sensor. For additional information regarding the TWM, the mounting system as well other collaborative efforts, please refer to Duvvuri et al. (2017).

All sensors were operated in a constant temperature anemometry (CTA) circuit, using the Dantec Streamline anemometer 90N10 with the 90C10 CTA module. The output data was recorded using a Dewetron DEWE-50-PCI-16 with a sampling frequency of 100 kHz.

# 3 Results

## 3.1 Subsonic Measurements

In order to quantify the free-stream turbulence level of the TWM, three different Mach numbers were investigated: $M = 0.3$, $M = 0.5$ and $M = 0.8$. Figure 3(a) shows the standard deviation of the mass flux, $\sqrt{\overline{(\rho u)'^2}}$, with respect to the mean mass flux, $\overline{\rho u}$. The standard deviation appears to increase monotonically for $M = 0.5$, while this trend is not observed at the other two Mach numbers. It was found that the turbulence intensity, $\frac{\sqrt{\overline{(\rho u)'^2}}}{\overline{\rho u}}$, is low in the free-stream for all three cases, ranging from $2-4\%$ for $M = 0.3$, $1-3\%$ for $M = 0.5$ and below $1.2\%$ for $M = 0.8$. An error analysis was performed to see if the inconsistency between curves was a result of error propagation. Errors stemmed from the sensors in the TWM as well as the fit of the calibration. However, accounting for the error did not modify the overall trend of the curves. In all of these measurements, fluctuations in $T_o$ were not accounted for. Although this is a reasonable approximation in the free-stream, these fluctuations can be significant in other conditions, especially with large values of mean shear.




(a) Standard Deviation of the Mass Flux versus Mean Mass Flux

(b) NSTAP/PIV Comparison at $M = 0.3$

Figure 3: a) Variation of the standard deviation of the mass flux data with respect to the mean mass flux for $M = 0.3$ (blue), $M = 0.5$ (red) and $M = 0.8$ (yellow). b) Comparison of the turbulence intensity found using PIV (blue) and the NSTAP (red) at $M = 0.3$. In order to transform $\sqrt{\overline{(\rho u)'^2}}$ into $\frac{\sqrt{\overline{u'^2}}}{\overline{u}}$, the data set was normalized by the mean mass flux, $\overline{\rho u}$.

The NSTAP data was also compared to PIV data. Planar PIV measurements were performed in a horizontal (streamwise-spanwise) plane near the NSTAP measurement location. The size of the field of view and the particle shift were optimized to resolve small-scale turbulent structures. Double frame images were recorded at 15 Hz. For the data presented in Figure 3(b), 500 statistically independent images were recorded and this was evaluated to be statistically converged. The standard deviation of the velocity fluctuations across the PIV field from all 500 images was less than $0.06\%$.

Unlike the coupling present between $\rho$ and $u$ when taking hot-wire measurements, $\rho$ and $u$ are no longer coupled when taking PIV measurements. Therefore, only the $M = 0.3$ data set was compared to PIV since this is the upper limit to the incompressible flow assumption. Figure 3(b) shows the turbulence intensity extracted using PIV $\left(\frac{\sqrt{\overline{u'^2}}}{\overline{u}}\right)$ and using the NSTAP $\left(\frac{\sqrt{\overline{(\rho u)'^2}}}{\overline{\rho u}}\right)$. While PIV results indicate that the turbulence intensity slightly decreases with increasing total pressure, the NSTAP data does not corroborate this trend. The offset between curves is expected due to the low-pass filtering of the data extracted with PIV. However, the reason for the discrepancy between trends is unclear with the current data set.

### 3.2 Supersonic Measurements

In order to investigate the heat transfer relationship governing the NSTAP, tests at a constant Mach number of $M = 2.0$ were performed at 3 different values of $\tau$ (0.9223, 0.7905 and 0.5904). During each run, the range of mass fluxes seen by the NSTAP was $285-320$ kg/m$^2$s (or equivalently a $Re$ range of $2.7\times10^7 - 3.2\times10^7$ m$^{-1}$). For all supersonic tests, data was collected with an unetched NSTAP. Figure 4 shows the calibration curves for each run. Interestingly, a linear relationship fits the data convincingly. This leads to the following heat transfer relationship:

$$E^2 = L(\tau) + M(\tau)\rho u \tag{4}$$

Although temperature does seem to affect the voltage drop across the wire, these effects can be accounted for by computing $L(\tau)$ and $M(\tau)$.

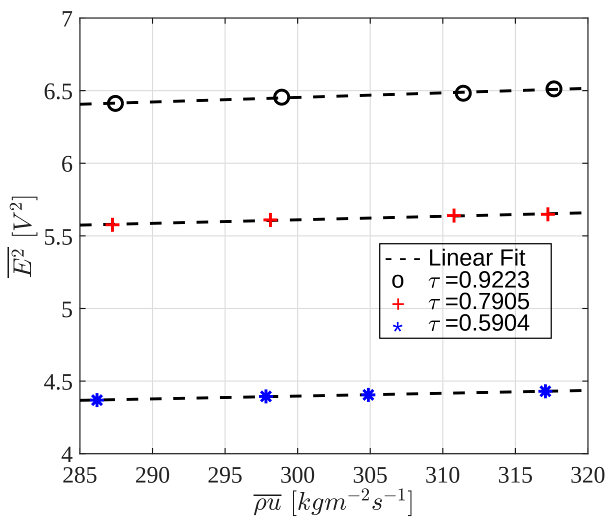



Figure 4: Plot showing the linear trend for all three overheat ratios. For each data set, a line was added for comparison and fits the data extremely well.

#### 3.2.1 Flow Regime: Possible Explanation for Linear Relationship

It is important to consider here the flow regime seen by the NSTAP. The ratio between the mean free path, $\lambda$, and a characteristic length scale, $l$, also known as the Knudsen number, $Kn$, is indicative of whether the continuum assumption holds or fails. $Kn$ can be defined as follows from kinetic theory:

$$Kn = \frac{\lambda}{l} = (\pi\gamma/2)^{1/2}\left(\frac{M}{Re}\right) \tag{5}$$

where the ratio of specific heats, $\gamma$, is 1.4 for diatomic gases such as air. According to this relationship, $Kn$ can increase by either increasing $M$ or by decreasing $Re$. In Dewey Jr (1965), it was found that while $Nu \sim Re^{1/2}$ in the continuum flow regime (where $\lambda << l$), it varies linearly with $Re$ in the free-molecule flow regime (where $\lambda > l$). This finding is informative and highly relevant for the NSTAP measurements. Given the small geometry of the sensing element and the high flow velocity, the NSTAP may be operating in a regime where the continuum assumption does not hold due to its small length scale and correspondingly, operating Reynolds number. Using the width of the sensing element as the length scale, $Kn = 0.05$ whereas using the thickness of the wire yields close to $Kn = 0.3$. The apparent linear relationship between $Nu$ and $Re$ may indicate that the thickness of the sensing element is the correct length scale governing this heat transfer. However, these results are preliminary and measurements over a larger range of mass fluxes should be acquired to validate this model.

## 4 Conclusion

In order to accurately measure turbulent compressible flows, high temporal and spatial resolutions are needed. For this, a variant of the nano-scale thermal anemometry probe was designed, manufactured and tested in the Trisonic Wind Tunnel Munich. First, subsonic data was collected and the free-stream turbulence level was computed at various Mach numbers. A clear trend of the turbulence level with $M$ was not found. Direct comparison with PIV measurements at $M = 0.3$ was inconclusive despite having accounted for error propagation, indicating that there must be unaccounted errors in either or both measurement techniques, which may be associated with density fluctuations. Second, the heat transfer relationship governing the NSTAP was determined by varying $Re$ and $\tau$, while keeping $M = 2.0$ constant. A linear trend was observed, which has previously been attributed to free-molecular flow. Linear calibration curves are always advantageous and thus future sensors could be designed to promote this behavior. Additional data over a larger range of mass fluxes must be collected to validate this proposed model.

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