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Equilibrium and Convergence in Personal Income Distribution? How European Countries Performed during a Phase of Huge Economic Turbulence (2004-2017)

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# **Equilibrium and convergence in personal income distribution? How European countries performed during a phase of huge economic turbulence (2004-2017)**

by

**Yanling Guo and Friedrich L. Sell**

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## **Abstract**

The authors set up a political economy equilibrium framework for personal income distribution. Located in status theory, their concept is able to explain what justifies a certain or optimal degree of inequality in the society. The authors present an empirical analysis of personal income distribution in 23 European countries. The time period covers the years before, during and after the great recession (2004-2017). Linear regressions, which make use of ex-post Gini coefficients, show that the hypothesis of the existence of an equilibrium value for the Gini-coefficient can be weakly confirmed, after controlling for a possible impact from the great recession as well as from EU membership.

**JEL Categories:** D31, D 60, D63, D71, D72, E62, H23

**Keywords:** Political Economy, Personal Income Distribution, Equilibrium, Convergence, Redistributive Policies, Great Recession, EU Membership

# **Equilibrium and convergence in personal income distribution?**

## **How European countries performed during a phase of huge economic turbulence (2004-2017)**

### **1. Introduction**

The main underlain hypothesis in this paper is the idea of equilibrium in personal income distribution. Already the popular economist Vilfredo Pareto (1895) held the view that the social distribution of personal incomes moves towards a stable equilibrium over time. He based his statement on the observation that the dispersion of personal incomes does not fluctuate, neither internationally or intertemporally. Much later, Hans Jürgen Ramser (1987) detected stationarity in the secondary distribution of personal incomes (net of government intervention with taxes and transfers), but not in the primary distribution of incomes (out of the market process). This empirical finding has been replicated recently (see Genc/ Miller/Rupasingha, 2011).

The existing skewness of (personal) income distribution can be interpreted as a display of social preferences, thereby implying that the preservation of a specific degree of income inequality in society is intentional (Blümle 1992, p. 224) and not arbitrary. Although distributional justice continues to be an important economic policy goal, it does not strictly focus on achieving perfect equitable income distribution. Both short- and long-run scenarios accept the unwarranted existence of a certain degree of inequitable distribution in the society (see Blümle 1992, p. 225). In reality, such an equilibrium will seldom be achieved to a full extent, but policy makers have good reasons to push towards the “steady state” and hence helping to reach convergence.

The paper hence introduces first - after a brief review of recent literature to our subject - a theoretical framework for equilibrium in income distribution. Thereafter, we will put our

hypothesis of equilibrium and convergence under a strong empirical test by considering a time span of extreme economic turbulence. This will be done through an empirical investigation of the period beginning in 2004, covering the deep financial market crisis (2007/2008), the worldwide economic crisis (2009–2010), and the full phase of the European debt crisis (2010–2017. The optimists would argue the crisis ended 2015, when the Grexit could again be avoided. We, in turn, are realists). The sample covers all those 23 EU countries for which we could find data of personal income distribution in the OECD database. At the end of the paper, we conclude with a discussion of our both theoretical and empirical results, study limitations of our analysis and look for possible next research steps.

## **2. A brief review of relevant literature**

In principle, there do exist two strands of literature relevant for our subject. While one perspective focuses on the relationship between *economic crisis* and personal income distribution, the other questions the existence of *convergence* in personal income distribution. However, there is no study available which connects these two items as our own paper does. For any of these research questions it is essential to address the Gini coefficient ex-post (1) or at least compare the Gini coefficient on market income (2) over time with the first. There exists one *pre-crisis* study (Van Kerm/Alperin 2013), wherein authors assume that the arrangement of countries in descending order of annual income inequality, for the period 2003–2007, puts “Portugal and Baltic states (such as Estonia, the authors) at the top and mostly Scandinavian countries (such as Finland and Denmark, the authors) at the bottom,” (p. 937). This result is a bit too unspecific. The papers of Dolls et al. (2011), De Beer (2012) and Kaitila (2013) deal with the issue of *economic crisis* and income distribution in Europe. Dolls et al. (2011) ran two controlled experiments (simulations) of macro shocks to income and employment and found that “both shocks lead to higher differences between the Gini coefficients based on equivalent

disposable and market income,” (ibid., p. 240). This applies to all of the 19 European countries considered by the authors. De Beer (2012), in our view, makes use of a far too narrow time span (2008-2009) to conclude that “the *economic crisis* has not so far led to a general widening of income disparities and a rise in poverty” (ibid, p. 23). However, there is no clarity over the approach that is employed by the author to calculate the Gini coefficients.

Employing a slightly different approach (calculating the Sigma *convergence*), Kaitila (2013) gained a result that is quite close to the findings of the current study. As per Kaitila’s findings, “For the EU-15 (a bit less so for the EU-27, the authors) we found that ... the national Gini coefficients have *converged* considerably during these (1995-2011, the authors) years,” (ibid., p. 14). Sell (2015, pp. 15-20) also achieved a very similar result. He states, “Globalization and possibly other forces linked to the revolution in communication and information technologies have contributed to an almost worldwide *convergence* in the distribution of personal incomes. More precisely, one can say that developing (developed) countries have become more equal (unequal),” (p.16).

It is surprising to notice that regressions are seldom run between GDP per capita and Gini coefficients of disposable income for huge projects on European inequalities, and thus their focus is restricted to the role of redistributive policies (Medgyesi/Toth 2009, p. 135; Paulus et.al 2009, p. 154). A clear step towards *convergence* was seemingly undetected between the period 2000 and 2005, a period wherein EU27 was without Cyprus, Malta, Slovakia and Luxembourg. As stated by a study, “the level of inequality at the beginning of the period does not seem to influence the direction and the magnitude of the change in inequality,” (Medgyesi/Toth 2009, p. 140). This result, however, is flawed. The discrepancy is attributed to the addition of countries to the EU, which turned from EU 15 into EU 25 only in 2004, and later into EU 27.

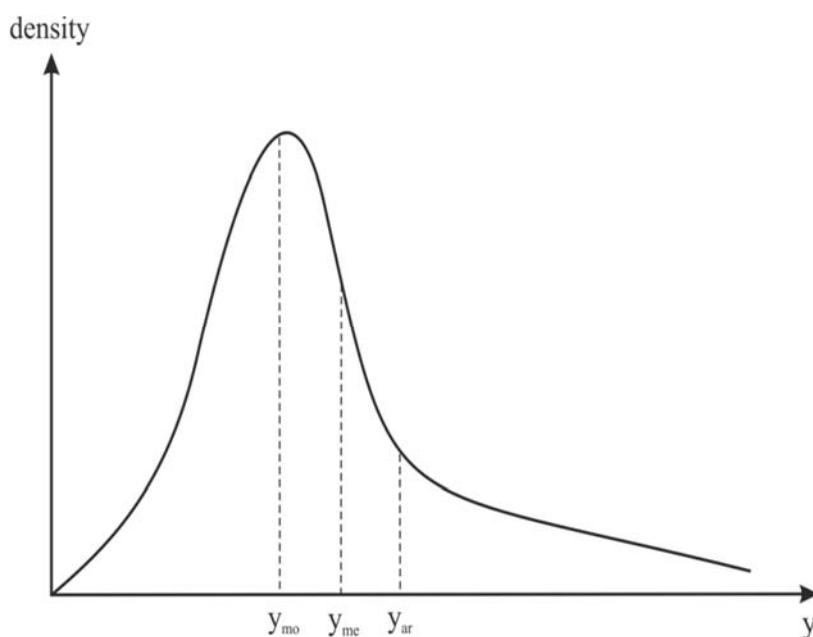
Hence, it remains to be shown how economic crisis, equilibrium and convergence in personal income distribution can be addressed scientifically in one single comprehensive approach. This is the task of the following sections.

### **Introducing a theoretical framework for equilibrium in income distribution**

Irrespective of the definition of income, economy in concern or the time period under consideration, it is surprising to see that the distribution of incomes is positively skewed (skewed to the right or steep on the left-hand). This has important consequences for the parameters of the density function. The maximum value of this function, which is called modus ( $y_{mo}$ ) and is the most frequent event, will usually be located at the left of the median ( $y_{me}$ ) and the latter, in turn, is located at the left of the arithmetic mean ( $y_{ar}$ ). The characteristics of this sort of density function are depicted in Figure 1.

**FIGURE 1**

#### **The time-invariant distribution pattern of personal incomes**



Source: Blümle (2005)



The consequence of this is far-reaching. According to Blümle (2005), a majority of economic agents will receive an income above the modus. Based on this observation, agents will have the impression of being well-paid, and hence their attitude towards a redistribution (the existing distribution) of incomes should be quite critical (benevolent). The density function of Figure 1 can be approximated, rather accurately, by a log-normal distribution of incomes:

$$Y = \exp(X) \text{ with } X = N(\mu, \sigma^2)$$

The expected or likewise average wage rate is then given by the following expression (see Beichelt and Montgomery 2003, pp. 46–8):

$$E(y) = y_a = \exp\left(\mu + \frac{1}{2}\sigma^2\right)$$

Taking the full differential of this expression from left to right yields:

$$dE(y) = dy_a = (d\mu + \sigma d\sigma) \exp\left(\mu + \frac{1}{2}\sigma^2\right)$$

**Proposition 1:** An increase in  $\sigma$  will shift the arithmetic mean to the right.

Furthermore, we have:

$$y_{mo} = \exp(\mu - \sigma^2)$$

Taking the full differential yields:

$$dy_{mo} = (d\mu - 2\sigma d\sigma) \exp(\mu - \sigma^2)$$

**Proposition 2:** An increase in  $\sigma$  will shift the modus to the left.

Finally, we have:

$$y_{me} = \exp(\mu)$$

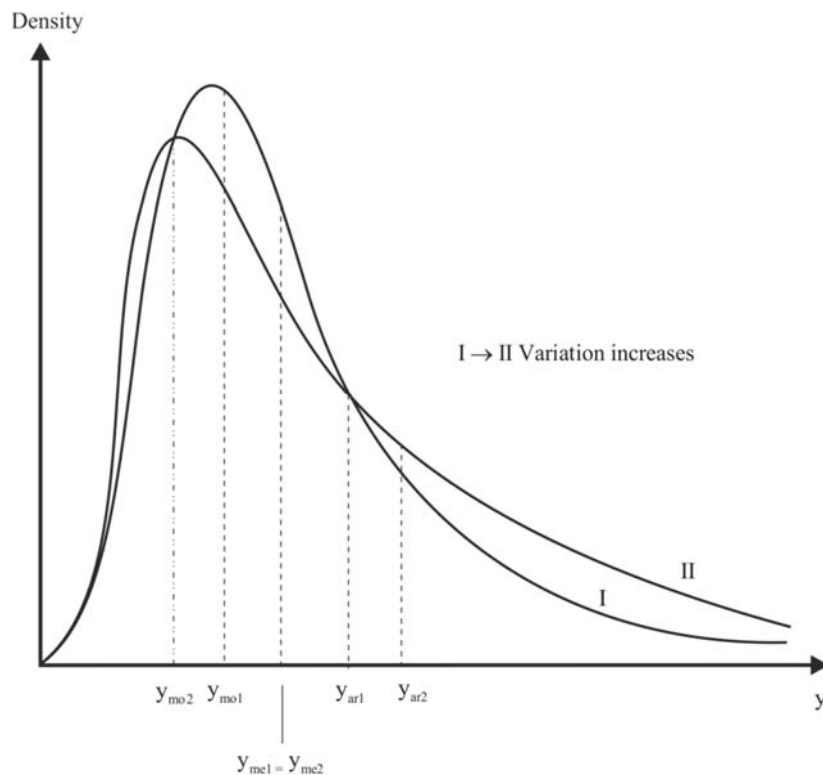
$$dy_{me} = \exp(\mu)$$

**Proposition 3:** An increase in  $\sigma$  will not affect the median

Furthermore, it holds for any  $\sigma^2 > 0$ :  $y_{mo} < y_{me} < y_{ar}$ .

**FIGURE 2**

**Increasing the standard deviation in the distribution of personal incomes**



Source: Sell (2015)

We now assume that an increase in inequality/a higher concentration of incomes is perceived by individual  $i$  as a loss of utility. The utility function of the individual  $i$  then reads as follows:

$$U_i = U_i(y_i - y_{mo}; \sigma).$$

Where,

$$\frac{\partial U_i}{\partial y_{mo}} < 0; \frac{\partial U_i}{\partial \sigma} < 0.$$

Assuming a law of diminishing increases of damage:

$$\frac{\partial^2 U_i}{\partial y_{mo}^2} > 0; \frac{\partial^2 U_i}{\partial \sigma^2} > 0$$

Hence, the corresponding iso-damage curves are concave. It is important to apply a budget constraint to determine an optimal solution, and such a budget constraint can be found in the properties of the log-normal distribution. Principally, an ideal log-normal distribution would offer a trade-off between a low dispersal of incomes and a low value of the modus. Therefore, the study presents a detailed analysis of the main characteristics of the log-normal distribution. The findings reveal that an increasing dispersion of incomes does not alter the median of the distribution (proposition 3), while the modus shifts to the left (proposition 2). The findings indicate a possible increase in the share of households that possess an income above the (new) modus.

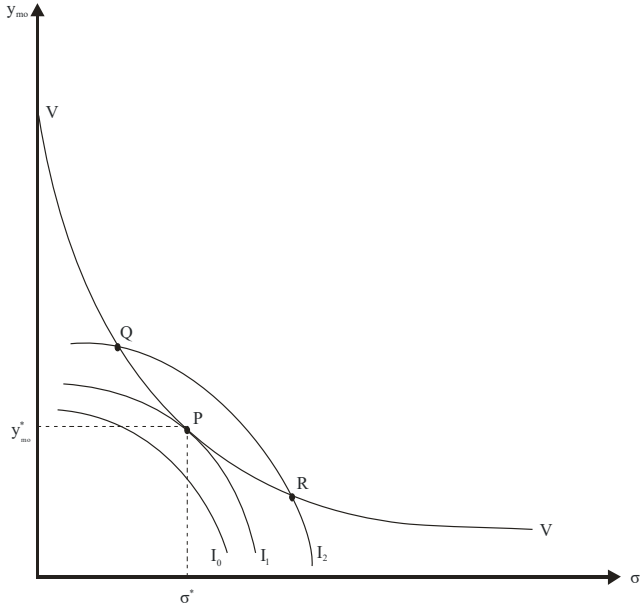
The following expression determines the mathematical solution for a Cobb-Douglas utility function in Figure 3:

$$U_i = \sqrt{y_i - y_{mo}} \sqrt{1/\sigma}$$

In the following diagram (Figure 3), we can determine equilibrium in personal income distribution. The axes have the modus ( $y_{mo}$ ) and the dispersion of incomes ( $\sigma$ ). The non-linear budget constraint, representing the log-normal distribution of incomes, is labeled as  $VV$ . This schedule is confronted with a troop of iso-damage curves ( $I_i$ ). The latter ones are concave to the origin of the coordinate system. The farther away these curves are located from the origin, the higher is the loss of utility of the individuals concerned. Point P signals towards a situation

where a preferably low iso-damage curve is tangential to  $VV$ . In a sense,  $P$  stands for equilibrium in income distribution. In comparison, points  $Q$  and  $R$  represent sub-optimal solutions. While  $Q$  and  $R$  fulfill the ‘budget constraint’ of the log-normal distribution, they are located on the less favorable iso-damage curve  $I_2$ .

**FIGURE 3**  
**Equilibrium in the distribution pattern of personal incomes**



Source: Sell (2015)

After learning the intuition of the optimal solution by graphical analysis, we now proceed to an explicit mathematical solution:

The individual  $i$  takes his own income,  $y_i$ , and the distributional parameters  $\mu$  and  $y_{m0}$  as given. Although the distributional parameter  $\sigma$  is also given to him, knowing that this parameter of income dispersion can be altered through redistributive policy by the government, he can deliberate on, which  $\sigma$  would be the best for him, and accordingly suggests his personal desired

level of income dispersion,  $\sigma^*_i$ , to the decision makers and thereby attempts to influence the finally determined level of income dispersion,  $\sigma^*$ , in his favor.<sup>1</sup>

First, we look at the personal desired level of income dispersion,  $\sigma^*_i$ . The maximization problem can be expressed as follows:

$$\max U_i \text{ s.t. } y_{mo} = \exp(\mu - \sigma^2)$$

Inserting the expression for  $U_i$ , we get:

$$\max \sqrt{y_i - y_{mo}} \sqrt{1/\sigma} \text{ s.t. } y_{mo} = \exp(\mu - \sigma^2)$$

The Lagrangian for this maximization problem is:

$$L_i = U_i - \lambda(y_{mo} - \exp(\mu - \sigma^2)) = \sqrt{y_i - y_{mo}} \sqrt{1/\sigma} - \lambda(y_{mo} - \exp(\mu - \sigma^2))$$

Taking the F.O.C. yields:

$$\frac{\partial L_i}{\partial \sigma} = -\frac{\sqrt{y_i - y_{mo}}}{2} \sigma^{-\frac{3}{2}} - 2\lambda \sigma \exp(\mu - \sigma^2) = 0 \quad (1)$$

$$\frac{\partial L_i}{\partial y_{mo}} = -\frac{\sqrt{1/\sigma}}{2} (y_i - y_{mo})^{-\frac{1}{2}} - \lambda = 0 \Rightarrow \lambda = -\frac{\sqrt{1/\sigma}}{2} (y_i - y_{mo})^{-\frac{1}{2}} \quad (2)$$

Inserting (2) in (1)  $\Rightarrow$

$$(y_i - y_{mo})^{-\frac{1}{2}} \exp(\mu - \sigma^2) \sqrt{\sigma} - \frac{\sqrt{y_i - y_{mo}}}{2} \sigma^{-\frac{3}{2}} = 0$$

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<sup>1</sup> Of course, hardly any one is able to calculate the exact value of his  $\sigma^*_i$ , and in the usual policy making process, the individuals cannot choose the final decision in a very exact manner. But suppose that everyone knows roughly what is best for him, and the final decision can roughly reflect the collective decisions, then our theoretical result is still a good approximation, around which the real decisions would center.

Substituting  $y_{mo}$  with  $\exp(\mu - \sigma^2) \Rightarrow$

$$(y_i - \exp(\mu - \sigma^2))^{\frac{1}{2}} \exp(\mu - \sigma^2) \sqrt{\sigma} - \frac{\sqrt{y_i - \exp(\mu - \sigma^2)}}{2} \sigma^{\frac{3}{2}} = 0$$

Multiplying both sides with  $(y_i - \exp(\mu - \sigma^2))^{\frac{1}{2}} \sigma^{\frac{3}{2}}$  yields:

$$\exp(\mu - \sigma^2) \sigma^2 - \frac{y_i - \exp(\mu - \sigma^2)}{2} = 0 \Rightarrow$$

$$\sigma^*_i = f(y_i, \mu)$$

Thus, the desired level of income dispersion by individual  $i$ ,  $\sigma^*_i$ , is uniquely determined by his own income,  $y_i$ , and the distributional parameter,  $\mu$ .

The finally determined overall level of income dispersion,  $\sigma^*$ , depends on all the individually desired levels,  $\sigma^*_i$ , and on the exact policy decision making process. In a process, in which all individual desires are assigned the same weight, the finally determined overall level of income dispersion,  $\sigma^*$ , is the parameter which maximizes the social welfare in form of aggregated utilities. Thus, we get the following optimization problem:

$$\max \sum_{i=1}^n U_i \quad \text{s. t. } y_{mo} = \exp(\mu - \sigma^2) \quad \text{and} \quad \bar{y}_a = \exp(\mu + \frac{1}{2} \sigma^2)$$

In the aggregated utility function,  $n$  denotes the number of individuals in the society. In addition to the known constraint,  $y_{mo} = \exp(\mu - \sigma^2)$ , we get a further constraint  $\bar{y}_a = \exp(\mu + \frac{1}{2} \sigma^2)$  where  $\bar{y}_a$  denotes the value of average income, which is constant, because we assume that the redistribution does not change the total income and henceforth the average income:

$$\sum_{i=1}^n U_i = \sum_{i=1}^n \sqrt{y_i - y_{mo}} \sqrt{1/\sigma} = \sqrt{1/\sigma} \sum_{i=1}^n \sqrt{y_i - y_{mo}}$$

Representing the utility function by a first order Taylor series approximation around  $y_a$ , we get:

$$\sum_{i=1}^n U_i = \sqrt{1/\sigma} \sum_{i=1}^n \left( \sqrt{\bar{y}_a - y_{mo}} + \frac{y_i - \bar{y}_a}{2\sqrt{\bar{y}_a - y_{mo}}} \right) = n\sqrt{1/\sigma} \sqrt{\bar{y}_a - y_{mo}}$$

The Lagrangian for the maximization problem is now:

$$L = n\sqrt{1/\sigma}\sqrt{\bar{y}_a - y_{mo}} - \lambda_1(y_{mo} - \exp(\mu - \sigma^2)) - \lambda_2(\bar{y}_a - \exp(\mu + \frac{1}{2}\sigma^2))$$

Taking the F.O.C. yields:

$$\frac{\partial L}{\partial \sigma} = -\frac{n}{2}\sigma^{-\frac{3}{2}}\sqrt{\bar{y}_a - y_{mo}} - 2\sigma\lambda_1 \exp(\mu - \sigma^2) + \sigma\lambda_2 \exp(\mu + \frac{1}{2}\sigma^2) = 0 \quad (3)$$

$$\frac{\partial L}{\partial y_{mo}} = -\frac{n}{2}\frac{\sqrt{1/\sigma}}{\sqrt{\bar{y}_a - y_{mo}}} - \lambda_1 = 0 \quad (4)$$

$$\frac{\partial L}{\partial \mu} = \lambda_1 \exp(\mu - \sigma^2) + \lambda_2 \exp(\mu + \frac{1}{2}\sigma^2) = 0 \quad (5)$$

From (5), we have  $\sigma\lambda_2 \exp(\mu + \frac{1}{2}\sigma^2) = -\sigma\lambda_1 \exp(\mu - \sigma^2)$ , substituting it in (3) yields:

$$\frac{\partial L}{\partial \sigma} = -\frac{n}{2}\sigma^{-\frac{3}{2}}\sqrt{\bar{y}_a - y_{mo}} - 3\sigma\lambda_1 \exp(\mu - \sigma^2) = 0 \quad (6)$$

From (4), we have  $\lambda_1 = -\frac{n}{2}\frac{\sqrt{1/\sigma}}{\sqrt{\bar{y}_a - y_{mo}}}$ , substituting it in (6) yields:

$$-\frac{n}{2}\sigma^{-\frac{3}{2}}\sqrt{\bar{y}_a - y_{mo}} + 3\sigma\frac{n}{2}\frac{\sqrt{1/\sigma}}{\sqrt{\bar{y}_a - y_{mo}}}\exp(\mu - \sigma^2) = -\frac{n}{2}\sigma^{-\frac{3}{2}}\sqrt{\bar{y}_a - y_{mo}} + 3\frac{n}{2}\frac{\sqrt{\sigma}}{\sqrt{\bar{y}_a - y_{mo}}}\exp(\mu - \sigma^2) = 0$$

Dividing both sides by  $n/2$ , then multiplying both sides with  $\sigma^{\frac{3}{2}}\sqrt{\bar{y}_a - y_{mo}}$  yields:

$$-(\bar{y}_a - y_{mo}) + 3\sigma^2 \exp(\mu - \sigma^2) = 0, \text{ which is equivalent to}$$

$$-(\bar{y}_a - y_{mo}) + 3\sigma^2 y_{mo} = 0, \text{ because } y_{mo} = \exp(\mu - \sigma^2)$$

Dividing both sides by  $y_{mo}$ , we get:

$$-(\bar{y}_a / y_{mo} - 1) + 3\sigma^2 = 0$$

Notice that both constraints imply that  $y_{mo} = \bar{y}_a \exp(-\frac{3}{2}\sigma^2)$  or  $\bar{y}_a / y_{mo} = \exp(\frac{3}{2}\sigma^2)$ , hence the above equation becomes:

$$-(\exp(\frac{3}{2}\sigma^2) - 1) + 3\sigma^2 = 0, \text{ which determines } \sigma^* \text{ at about } 0.9.$$

### 3. Empirical Research

In the last section, we set up a model, which shows that the (collective) choice of the preferred variance of income distribution results from a trade-off between the preference for one's own above-modus income and the preference for a more equal income distribution of the society. A testable hypothesis derived from the model is that there exists a long-run equilibrium value of income distribution variance, towards which the society converges.

To conduct the empirical test, however, we have first to resolve two issues. First, the empirical data often contains the Gini-coefficient instead of the variance of the income distribution. Second, it sounds counter-intuitive that societies of different histories and development stages should all converge to "the" optimal variance of income distribution, which, as we will see later, is also not supported by the data.

The first issue can be easily solved due to the assumed log-normal distribution of incomes, which captures several real-world features (a left-steep/right-skewed income distribution with a modus lower than the median and a median lower than the mean) and thus serves as a good approximation of the real-world data. Given the log-normal distribution as a fairly good approximation of the empirical distribution of incomes, it is a well-known fact that the Gini-coefficient is a monotonically increasing function of the variance or its root, the standard deviation  $\sigma$ .<sup>2</sup> Thus, the convergence to a particular value of variance is equivalent to the convergence towards a particular Gini-coefficient of the income distribution.

The second issue can also be solved easily by introducing a preference parameter into our basis model. The utility function becomes:

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<sup>2</sup> More precisely,  $Gini = 2\Phi(\sigma/\sqrt{2}) - 1$  with  $\Phi(x)$  being the standard normal distribution. This formula can be found in any advanced econometric textbook about inequality statistics.



$$U_i = \sqrt{y_i - y_{mo}} \left( \sqrt{1/\sigma} \right)^\gamma$$

The now introduced parameter  $\gamma$  stands for the weight, which the individuals of a society assign to the preference for a more equal income distribution (or against a higher  $\sigma$ ), relative to the preference for a larger above-modus own income. The larger  $\gamma$  is, the lower will be the equilibrium standard deviation,  $\sigma^*$ .<sup>3</sup> While it is possible that  $\sigma^*$  does not only vary across regions, but also changes over time due to shifts in preferences, it should remain relatively stable, such that an equilibrium state of  $\sigma^*$  can still be observed when the underlying force driving the change is absent.

To test our theory, we use all available data for the 23 EU countries which we can find in the OECD data base.<sup>4</sup> The time period for which we can find Gini-coefficients for the EU countries covers 2004-2017. The 23 EU countries in alphabetic order are Austria, Belgium, Czech Rep, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Latvia, Lithuania, Luxembourg, Netherlands, Poland, Portugal, Slovakia, Slovenia, Spain, Sweden and UK. Most of them were already EU members in 2004 and some just joined in in this year.

Two features characterize our sample: the countries under consideration (EU members) and the time span (2004-2017) chosen. EU membership implies several important aspects: strong trade integration and (but to a less extent) the integration of factor markets among the countries investigated. The latter points at a convergence of tradable goods prices and of factor prices used in the production of tradeable goods. This effect is most likely the stronger, the longer the membership in the EU of the respective country. Also, one may guess that a harmonization of the phases of the business cycle with regard to other countries will occur with greater probability, the longer EU membership prevails. As a consequence, the pattern of shifts between profit income and labor income will become more similar. Moreover, becoming a member in the EU implies the adhesion to the catalogue of rules and regulations incorporated in the “acquis communautaire”. Or in other words: the adhesion to the principles of the European welfare state and to the way, the welfare state corrects the concentration of market incomes (Gini ex-ante) by taxes and transfers in order to achieve a socially acceptable inequity of incomes (Gini ex-post).

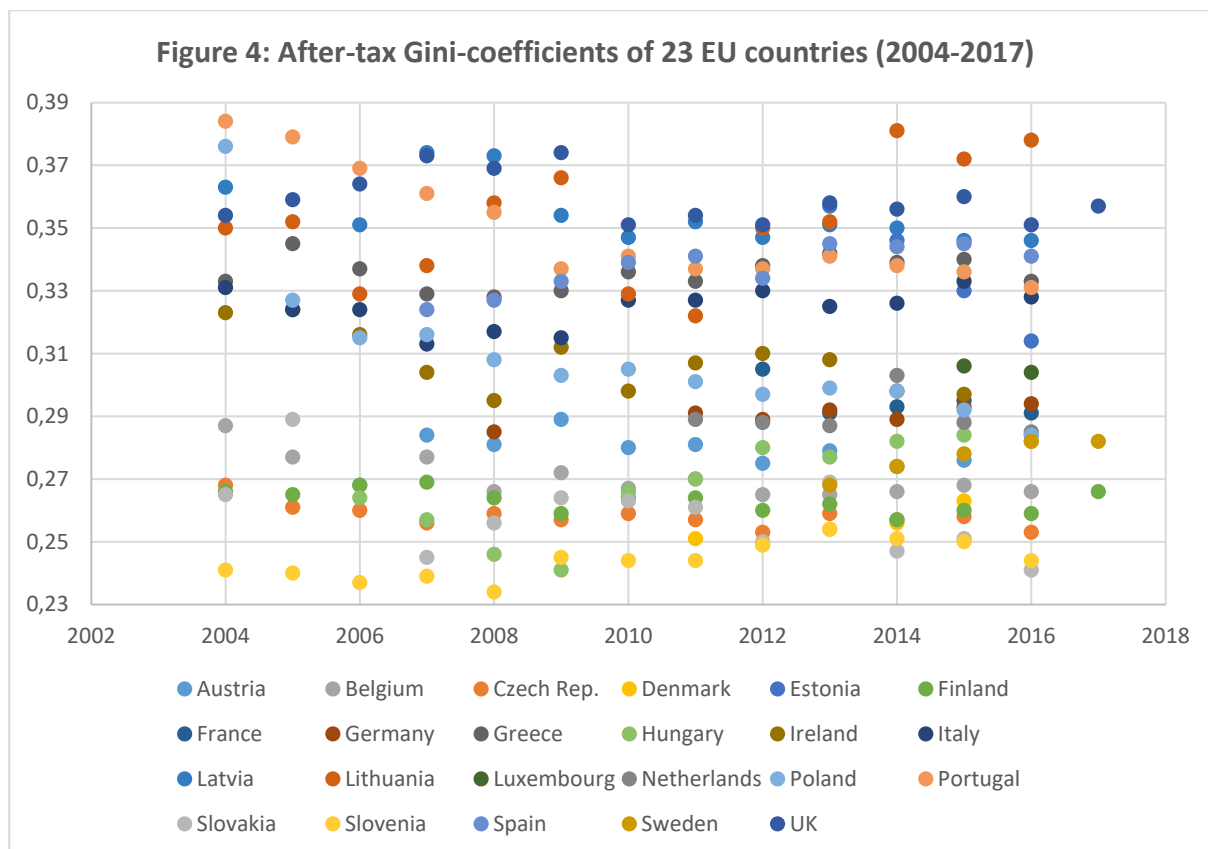
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<sup>3</sup> See appendix.

<sup>4</sup> The 23 EU countries include UK, which was a member during the entire time span. Not included are Bulgaria, Croatia, Cyprus, Malta and Romania, which are absent in the OECD data base.

The second important feature of our database is the time period under investigation. It encompasses severe economic crises on the world scale level (2008-2010), but also on the European level (2010-2015). In both cases, governments pursued large public expenditure programmes to fight and overcome the crises. This presumably hindered policy makers – to some degree - to correct the income distribution of market incomes according to their preferences and “as usual”: temporarily, hence, one may assume Gini coefficients ex-post to have reached some distance to the desired values or, likewise, to equilibrium in income distribution. As the public expenditure programmes were “uni sono” of Keynesian nature, one may further conclude that their impact on distribution (of both personal incomes and of factor incomes) should have been quite similar among the countries at stake, so that a “convergence in divergence” may be observed in the data.

In figure 4, we plot all the time series: On the first glance, figure 4 seems to contradict our theory by showing some EU countries with rising, some with falling, and again others with relatively constant Gini-coefficients. This optic perception is largely confirmed by the regression results summarized in table 1, in which the Gini-coefficients of each country is regressed on time and quite a few countries display a statistically significant slope -- though here we must pay attention that some of them have a close to zero slope which is only statistically significant due to the small variation in the data. But even countries with large and significant slope do not necessarily contradict the theory, because the 23 EU countries differ in the length (in full years) of their membership, ranging from 0 to 58. It is plausible to assume that new members rather move towards the new equilibrium, which is probably the group mean, than old members. Indeed, figure 5, in which the data is not simply plotted against the time horizon, but to the length of membership, seems to confirm this assumption.

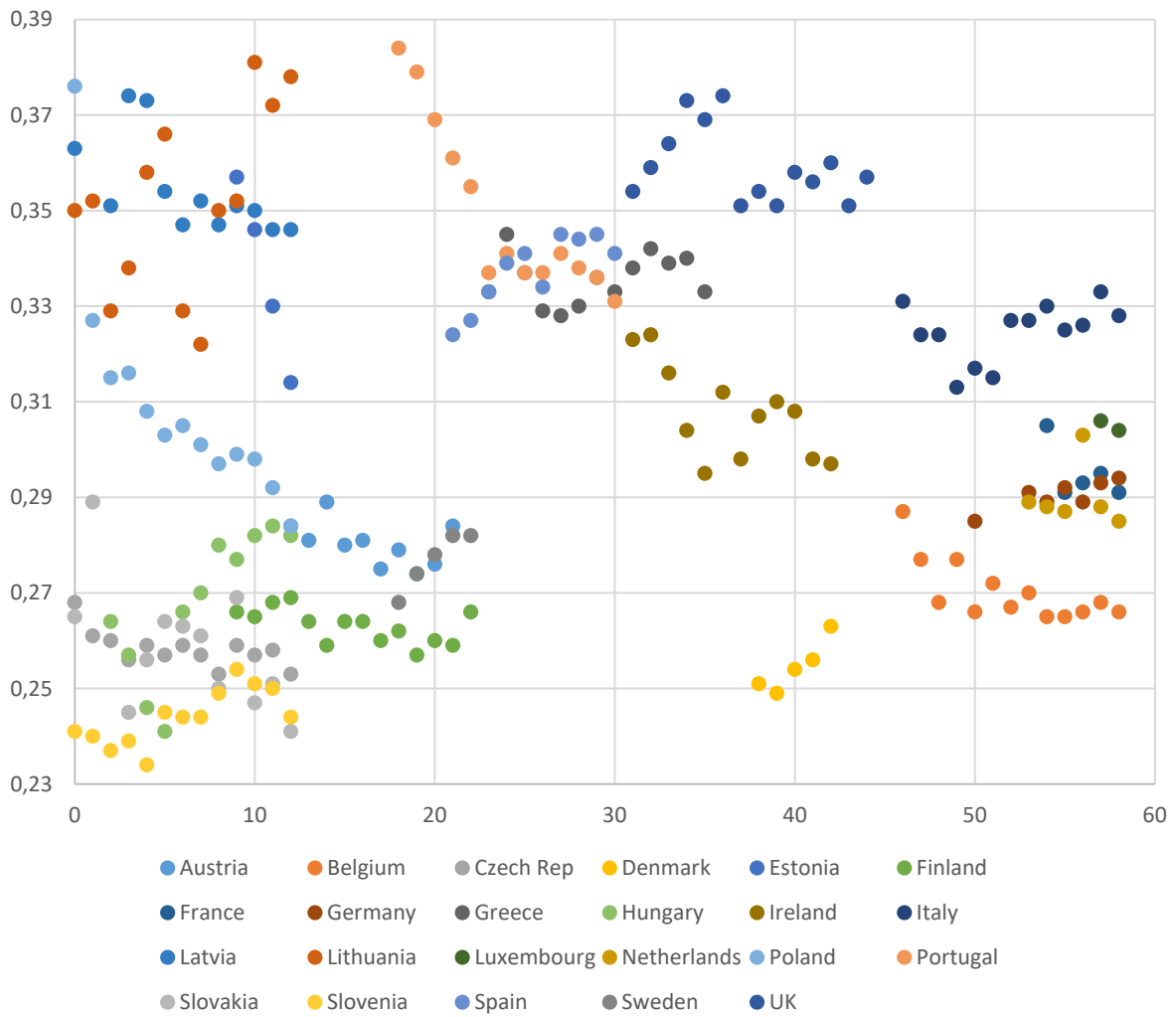


**Table 1: OLS regression of after-tax Gini-coefficients (23 EU countries 2004-2017)**

Country	n	slope	Standard deviation	t-value	p-value
Austria	10	-0.00072121	0.00047544	-1.51693934	0.1677561
Belgium	13	-0.0012033	0.00034714	-3.46635929	0.0052737
Czech Rep	13	-0.00065934	0.00021575	-3.05607962	0.0109282
Denmark	5	0.0031	0.00083865	3.69641815	0.03436319
Estonia	4	-0.0145	0.00086603	-16.7431578	0.00354821
Finland	14	-0.0005033	0.00020551	-2.44899944	0.03064925
France	5	-0.0024	0.00161658	-1.48461498	0.23430944
Germany	7	0.00096382	0.00026607	3.62247763	0.01517997
Greece	13	0.00024725	0.00039347	0.62838666	0.54257778

Hungary	11	0.00356364	0.00091782	3.88273206	0.00371589
Ireland	12	-0.00183217	0.00064268	-2.85083659	0.01722537
Italy	13	0.0005989	0.00043898	1.36429375	0.19973625
Latvia	13	-0.00252198	0.00078811	-3.20001754	0.00845441
Lithuania	13	0.00251648	0.00125603	2.00352319	0.07037776
Luxembourg	2	-0.002	0	65535	#ZAHL!
Netherlands	6	-0.00011429	0.00173934	-0.06570634	0.95076452
Poland	13	-0.00478022	0.00103244	-4.63004199	0.00072836
Portugal	13	-0.00413736	0.00061505	-6.72684721	3.2567E-05
Slovakia	12	-0.00195951	0.00087567	-2.23774359	0.04919218
Slovenia	13	0.00108791	0.00031177	3.48948856	0.00506312
Spain	10	0.00209091	0.00046939	4.45448769	0.00212658
Sweden	5	0.0036	0.00061101	5.89188304	0.00975908
UK	14	-0.00071429	0.00050097	-1.42581349	0.17941387

Figure 5: After-tax Gini-coefficients vs. length of EU membership



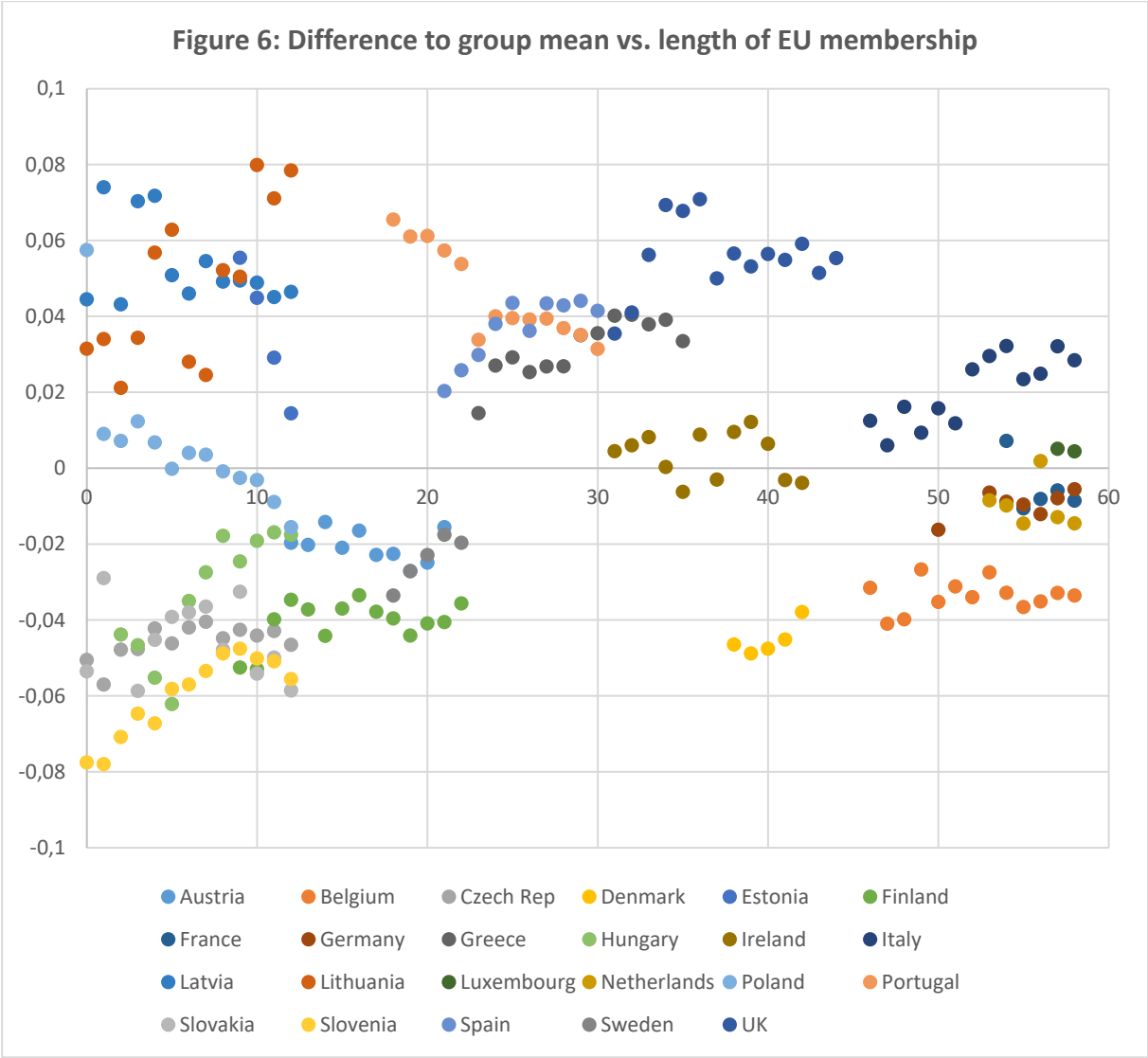


Figure 5 depicts the same Gini-coefficients as figure 4 but confronts them with the length of EU membership instead, which is calculated as the current year minus the year joining the EU. For instance, Austria joined EU in 1995, then in 2007 its length of membership is 12. Hungary joined EU in 2004, then in 2007 its length of membership is 3. Optically, it is like showing figure 4 again, but with time series of newer member countries moved to the left and older member countries to the right, according to the length of their EU membership. After this rearrangement, it becomes visible that large changes in Gini-coefficients rather occurs in the newer member countries (on the left), which is compatible with the assumption that joining the EU can change the equilibrium value of the country. Moreover, it is plausible to assume that this new equilibrium would be somewhere near the group mean, which is the case for the most countries.

To make things look even clearer, we also present figure 6, which has the same x-axis as figure 5, but with distance to the group mean on the y-axis, (calculated as the Gini-coefficient minus the group mean,) instead of the Gini-coefficient itself. It is clearly visible that there is a narrowing in the bandwidth with increasing length of EU membership, which means that with longer EU membership, the Gini-coefficients of the member countries become on average closer to each other. In other words, most countries have got Gini-coefficients closer to the group mean, some fluctuate around the mean, some divert, but the magnitude of diversion is smaller than the magnitude of the conversion of their peers.

Of course, there may be other factors which could change the equilibrium. A further candidate is the financial crisis starting in 2007. While it is easier to conceive that the EU membership as an integration process can alter or assimilate the collective preference for a more equal income distribution of the society vs. a larger above-modus own income, it needs some explanation for why the financial crisis could also affect the after-tax Gini-coefficient. It is plausible to assume that the financial crisis as an economic event with large impact can affect the pre-tax Gini-coefficients as a result of market activities. It is also plausible to assume that the redistributive correction may not keep pace with the change in pre-tax Gini-coefficients. Thus, reflected in the data, we would also see a short time diversion from the equilibrium value of after-tax Gini-coefficients, though this effect would unlikely change the long-run equilibrium, unless the lesson learned from the financial crisis would also induce a change in the collective preferences.

In the following, we run a pooled regression in the form:

$$\delta = c + X\beta, \text{ where}$$

$$\delta_t = \ln(Gini_t) - \ln(Gini_{t-1})$$

If the basic model were true, then both  $c$  and  $\beta$  would be not significantly different from 0.

A quick look into the values of  $\delta$  seems to confirm this hypothesis, since  $\delta$  has mean -0.003 and standard deviation of 0.03. However, here we did not check for  $\beta$ , and this first conclusion is clearly in contrast to figure 4 and table 1, where in some countries there is a significant and large movement in  $\sigma$  and hence a significant and sizable country specific  $\delta$ . In fact, a quick regression of  $\delta$  on its own predecessor shows a significant parameter with t-value of -4, which is clearly in contrast to the basic model, which predicts  $\delta$  to be a white noise and independent of any other variable including its own predecessor.

The extended model with a parameter  $\gamma$ , which can change over time but only if there is a plausible reason for a significant change in the collective preference, also predicts  $c=0$ , but it allows for non-zero  $\beta$ s for  $X$ s standing for the possible reasons. In this paper, we consider the financial crisis and the joining of EU as two candidate reasons, though we do not rule out that there may be other reasons which escaped our attention.

To model the impact of the financial crisis, we include in  $X$  a dummy variable, which equals to one when the year is 2008. We are not sure if  $\delta$  would further change in the aftermath of the financial crisis or it only jumps in 2008, so we also include the logarithm of the length of financial crisis, which is  $\ln(\text{year}-2007)$  if  $\text{year}>2007$  and 0 otherwise. These two variables can never be both non-zero at the same time and with them we want to model that the start of the financial crisis may have a special, jump effect, while all years after it (including the phases of the worldwide economic crisis and the European debt crisis) can be modeled the same way following a logarithm function, i.e., the effect may fade over time and later years distant from 2008 would have the same (no) effect.

The modeling of the impact of joining of the EU is more complicated. As figure 6 suggests, the change in the Gini-coefficient does not only depend on how long a country is in the EU, but also on how far it is from the group mean. Because we are not sure about the functional form, we do not only include the time effect including a dummy and a log of length, analogous to the way we model the financial crisis, and the variable “distance previous”, calculated as  $\text{Gini} - \text{groupMean}(\text{Gini})$  in the previous year, but also the interaction terms,  $\text{distance} * \text{dummy}$ , called “distance start previous”, and  $\text{distance} * \ln(\text{length})$ , called “distance length previous”. The data shows that the  $\delta$ s are serially negatively correlated, besides, it may depend not only on the previous but also on the current deviation from its group mean, hence we also include the current terms of “distance” and interacted it the same way. Because we don’t want the possible uncentered data to distort the estimation of  $c$ , which is the variable of central interest in this paper, we centered all variables by subtracting their means, and the centered variables are denoted with an asterisk.

In table 2 we summarize all independent variables except the constant term.

Table 2: the definition of (independent) variables	
variable	definition



distance previous*	the centered distance to group mean of Gini in t-1
distance start prev*	the centered product of “distance previous” with ”EU dummy”
distance length prev*	the centered product of “distance previous” with ”EU log”
distance*	the centered distance to group mean of Gini in t
distance start*	the centered product of “distance” with ”EU dummy”
distance length*	the centered product of “distance” with ”EU log”
EU dummy*	the centered dummy for the year of joining EU
EU log*	the centered logarithm of length of being in the EU
FC dummy*	the centered dummy for the year of financial crisis (2008)
FC log*	the centered logarithm of length of entering financial crisis

Table 3 summarizes the regression result.

Table 3: the pooled regression result				
variable	estimate	sd	t-value	p-value
constant	-0.00308479	0.00068747	-4.48717589	1.2166E-05
distance previous*	-3.60915023	0.19322145	-18.6788282	1.0042E-45
distance start prev*	0.66247003	0.25314668	2.61694142	0.00954995
distance length prev*	0.18239447	0.07615187	2.39514107	0.01753642
distance*	3.67870821	0.19987721	18.4048406	6.5404E-45
distance start*	-0.76819633	0.26074203	-2.94619294	0.00359896
distance length*	-0.20543564	0.07796549	-2.63495618	0.00907399
FC dummy*	0.00403563	0.00292981	1.3774367	0.16991678

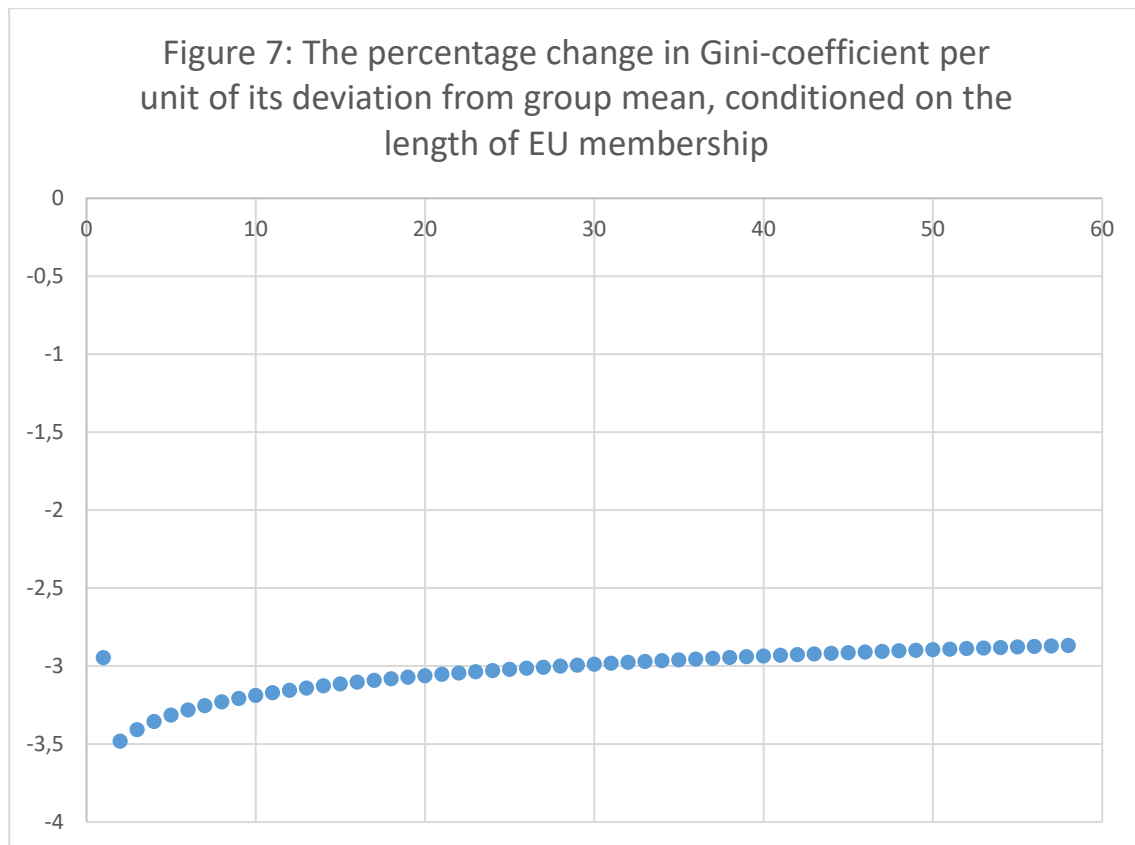
FC log*	0.00680377	0.00100938	6.7405549	1.65E-10
EU dummy*	0.01406225	0.00482321	2.91553551	0.00395588
EU log*	0.00087563	0.00082192	1.06534363	0.28800489

The result is mixed. The constant term is statistically significant. However, the magnitude is very small, thus, we cannot reject the hypothesis of the existence of an equilibrium value of  $\sigma$ , which can be subject to change. Here we only include two possible sources of the change, there may be some other factors which we didn't consider. More data could bring new insight.

Both sources considered here show significant impact. In case of the financial crisis, the impact from the aftermath is more significant, with a p-value close to zero, while the "jump" effect in 2008 is not very significant, with a p-value of 0.17. Possibly it takes some time for the impact to become effective. The impact of the EU membership alone is significant in the year of joining the EU, and not very significant afterwards. The impact from the distance to the group mean of the Gini-coefficients is not only significant, but also sizable. Here we only look at the estimates for the variables in the previous period, since the variables in the current period are only included to account for the serial negative correlation in  $\delta$ . A one percentage point deviation of the own Gini-coefficient above (below) the group mean in period t-1 will lead to a decrease (increase) of the Gini-coefficient by about 3 per cent from t-1 to t.<sup>5</sup> Recall that we allowed for time dependency of this effect by interacting it with the time effect variables, which means that this effect will fade from the second year on. After adding all effects, we can see that there is a jump effect in the first year of joining EU and the effect afterwards is approximated by a logarithmic function. Figure 7 depicts the percentage change in Gini-coefficient, namely  $\delta$ , as a multiple of the deviation from the group mean, plotted against the length of EU membership.

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<sup>5</sup> From t-1 to t, the base effect is -3.61, plus a time jump effect of 0.66, makes a total effect of about -3 in the first year.



#### 4. Discussion, conclusions and the scope for future research

On the background of intense discussion in Europe on the increasing skewness in the distribution of incomes and wealth, the study puts forward an equilibrium concept of personal income distribution. After controlling for the impact from the financial crisis (and its aftermath) and the EU membership (and a possible assimilation of the Gini-coefficients), we get a near zero change in the Gini-coefficient, which is only significant due to the precision of the estimation. These outcomes would not only support the notion of equilibrium in the distribution of personal incomes, but also further a concept of convergence in the distribution of incomes.

The limitations of our study are clearly related to the availability of consistent and comparable data of statistical moments of personal income distribution, such as the modus of incomes. As a consequence, we were unable to test directly all of the implications of the theoretical model.

Future analysis of personal income distribution in Europe could possibly intend to deepen our knowledge about the convergence stimulating effects of economic crises or change in

institutional setting and also interconnect the analysis of personal income distribution with the development of macroeconomic shares of total income (profits and wages).

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## Appendix

The aggregated utility function becomes to

$$\sum_{i=1}^n U_i = n\sqrt{1/\sigma^\gamma} \sqrt{\bar{y}_a - y_{mo}}$$

Inserting  $y_{mo} = \exp(\mu - \sigma^2)$  and  $\bar{y}_a = \exp(\mu + \frac{1}{2}\sigma^2)$  yields:

$$\sum_{i=1}^n U_i = n\sqrt{1/\sigma^\gamma} \sqrt{\bar{y}_a (1 - \exp(-\frac{3}{2}\sigma^2))}$$

Note that  $n$  and  $\bar{y}_a$  are both positive constants,

$$\max \sum_{i=1}^n U_i \equiv \max \sqrt{1/\sigma^\gamma} \sqrt{1 - \exp(-\frac{3}{2}\sigma^2)} \equiv \max \frac{1 - \exp(-\frac{3}{2}\sigma^2)}{\sigma^\gamma}$$

Taking the F.O.C. yields:

$$-\gamma\sigma^{-\gamma-1} (1 - \exp(-\frac{3}{2}\sigma^2)) + \sigma^{-\gamma} \exp(-\frac{3}{2}\sigma^2) 3\sigma = 0$$

multiplying both sides with  $\sigma^{\gamma+1} \exp(\frac{3}{2}\sigma^2)$  yields:

$$-\gamma(\exp(\frac{3}{2}\sigma^2) - 1) + 3\sigma^2 = 0$$

Denote the value of F.O.C. at equilibrium as  $F$ , then

$$\frac{d\sigma^*}{d\gamma} = -\frac{\partial F / \partial \gamma}{\partial F / \partial \sigma} = \frac{\exp(\frac{3}{2}\sigma^2) - 1}{\partial F / \partial \sigma}$$

Because  $\sigma^2 > 0$ , the numerator is positive, because it is evaluated at the maximum, the denominator is negative, thus,  $\sigma^*$  decreases when  $\gamma$  increases:  $\frac{d\sigma^*}{d\gamma} < 0$ .

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