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Turbulent Pair Diffusion of Inertial Particles Using Kinematic Simulations

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Abstract

Turbulent inertial particle pair diffusion is investistigated in the limit of Stoke's drag using Kinematic Simulations. For large Stokes number, $St \gg 1$, the inertia dominates and we observe ballistic motion for inertial pair separation. For small Stokes number, $St < 1$, the turbulent energy dominates the diffusion asymptotes to the fluid pair diffusion.

1 Introduction

Understanding the transport processes governing inertial particle motion is important because there are many aplications in industry and natural contexts, from dust storms and pollens, to bubbles, and hail, Calzavarini et al. (2008); Falkovich and Pumir (2007); Shaw (2003); Sofiev and Bermann (2013); Toschi and Bodenschatz (2009). The motion of groups of particles, such as dust storms, can often be related to the relative motion of two particles, or pair diffusion.

The transport equations that describe the motion of individual inertial particles are not fully developed yet, although simplified descriptions in specific contexts have been proposed by Maxey and Riley (1983). The suspended particles have finite size, and density different from that of the carrier fluid, and as a consequence the interactions between the particle and the underlying flow structures plays an important role; heavy particles are expelled out of vortical structures, while light particles tend to concentrate in their cores, leading to preferential concentration and the formation of strong inhomogeneities in the particle spatial distribution Qureshi et al. (2007).

Richardson (1926) proposed a theory of fluid partice pair diffusion based upon the idea of a scale dependent pair coefficient, $K_f(l)$, where l is the distance between two particles, and on the locality hypothesis in which only energy in the turbulent scales which are of a similar size to the pair separation l itself is effective in further increasing the pair separation. This yields the 4/3-scaling for the diffusion coefficient, $K_f \sim l^{4/3}$. Obukhov Obukhov (1941) showed that this is equivalent to $\sigma_l^2 = \langle l^2 \rangle \sim t^3$ known as the t^3- regime. $\langle \cdot \rangle$ is the ensemble average. In the ensuing discussions, we follow the usual convention of replacing the scaling on l with its rms value, i.e. $l \sim \sigma_l$.

However, a new non-local theory of turbulent fluid particle pair diffusion has been proposed in Malik (2018a,b) in which both local and non-local processes govern pair diffusion in high Reynolds number turbulence. For Kolomogrov turbulence, $E(k) \sim k^{-5/3}$, in the limit of very large inertial subrange the theory predicts the scalings, $K_f \sim \sigma_l^{1.53}$.

A key question is, do the ideas of locality and non-locality extend to inertia particle pair diffusion ? Inertial particle diffusion has seen growing interest recently, Bec et al. (2010b); Chang et al. (2015); Bec et al. (2010b); Chang et al. (2015); Bragg et al. (2016); Bragg (2017); Bec et al. (2010a); Gustavsson and Mehlig (2011); Gustavsson et al. (2014); D and R (2014). However, none of these works specifically address the the problem of non-local turbulent transport processes.

Here, we investigate the impact of local and non-local turbulent transport processes on inertial particle pair diffusion inside the inertial subrange. To address this problem, we use Kinematic Simulations (KS) which can generate very large inertial subranges.

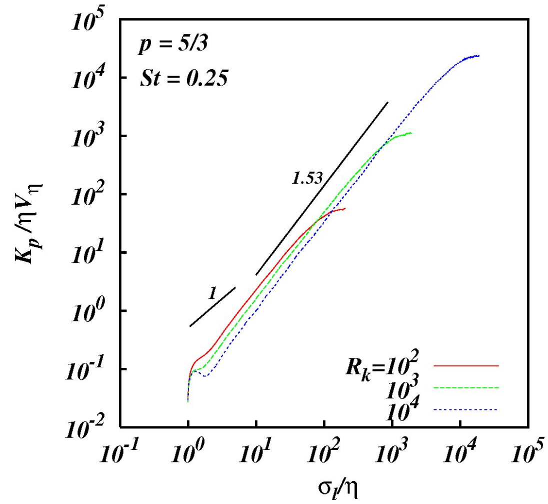

Figure 1: Log-log of the inertial pair diffusion coefficient $K_p/\eta v_\eta$ against the rms pair separation σ_l/η from KS simulations with energy spectrum $E(k) \sim k^{-5/3}$, for inertial subranges of size $R_k = 10^2$, 10^3, and 10^4. Here, the particle Stokes number is $St = 0.25$.

2 Inertial particles

We investigate numerically turbulent pair diffusion of inertial particles in high Reynolds number turbulence in the limit of large inertial subrange, $R_k = k_\eta/k_1 \to \infty$, and in the Stokes drag limit. The particle trajectory is then obtained by integrating the coupled tansport equations for the particle velocity $\vec{v}(\vec{x},t)$ in a fluid flow $\vec{u}(\vec{x},t)$ at the location and time $(\vec{x},t)$,

$$\frac{d\vec{x}}{dt} = \vec{v}(t) \quad (1)$$

$$\frac{d\vec{v}}{dt} = -\frac{1}{\tau_p}\left(\vec{v}(t) - \vec{u}(\vec{x},t)\right) \quad (2)$$

τ_p is the particle relaxation time which accounts for the particle inertia. The global Stokes number is,

$$St = \frac{\tau_p}{t_\eta} \quad (3)$$

where $t_\eta \sim \varepsilon^{-1/3}\eta^{2/3}$ is the Kolmogorov time scale of the turbulence. ε is the rate of energy dissipation per unit mass, and η is the Kolmogorov length scale. A local Stoke's number depending on the local separation can also be defined,

$$St(l) = \frac{\tau_p}{t_l} \quad (4)$$

where $t_l \sim \varepsilon^{-1/3} l^{2/3}$ is the turbulence time scale at lengths scale $\sim 1/l$.

We consider an effective point source release of inertial particles and assume that inertial pair diffusion can also be described by a diffusion equation with a scale dependent diffusion coefficient. In the limit to Stoke's drag, the diffusion coefficient will then be a function of two variables, $K_p = K_p(l, St)$.

For small separations, the particle inertia is expected to dominate over the small scale turbulent energy, thus we should observe ballistic motion, and K_p should be linear in the separation,

$$K_p(l,St) \sim \sigma_l^1, \qquad \sigma_l \ll \sigma_l^* \quad (5)$$

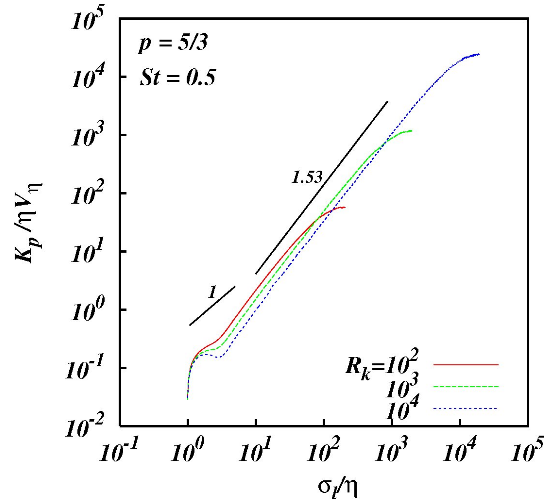

Figure 2: Same as Figure 1, except here the particle Stokes number is $St = 0.5$.

where σ_l^* is the scale where the inertia and turbulent energies are balanced, which is expected to occur when the timescales are equal, i.e. when $St(\sigma_l^*) = 1$, so that $t_{\sigma_l^*} = \tau_p$, Bec et al. (2010b).

At very large times, the turbulent energy is expected to be dominant, and we expect the inertia pair diffusion to asymptote towards the fluid pair diffusion provided that the inertial subrange is big enough for the pair separation to still remain within the subrange. Thus,

$$K_p(l,St) \quad \to K_f(l) \sim \quad \sigma_l^{1.53}, \qquad \sigma_l \gg \sigma_l^*. \tag{6}$$

3 Kinematic Simulations

In KS one specifies the second order Eulerian structure function through the power spectrum, like $E(k) \sim k^{-5/3}$, $k_1 \leq k \leq k_\eta$, Kraichnan (1970); Fung et al. (1992); Malik (2017). KS can generate inertial subranges sufficiently large to test pair diffusion scaling laws over extended inertial subranges. KS generates turbulent-like non-Markovian particle trajectories by releasing particles in flow fields which are prescribed as sums of energy-weighted random Fourier modes. By construction, the velocity fields are incompressible and the energy is distributed among the different modes by a prescribed Eulerian energy spectrum, $E(k)$. The essential idea behind KS is that the flow structures in it - eddying, straining, and streaming zones - are similar to those observed in turbulent flows, although not precisely the same, which is sufficient to generate turbulent-like particle trajectories.

KS has been used to examine single particle diffusion Turfus and Hunt (1987); Murray et al. (2016), and pair diffusion Fung et al. (1992), Murray et al. (1996), Fung and Vassilicos (1998), Malik and Vassilicos (1999), Nicolleau and Nowakowski (2011). KS has also been used in studies of turbulent diffusion of inertial particles Meneguz and Reeks (2011), Farhan et al. (2015). Meneguz & Reeks Meneguz and Reeks (2011) found that the statistics of the inertial particle segregation in KS generated flow fields for statistically homogeneous isotropic flow fields are similar to those generated by DNS.

KS pair diffusion statistics have been found to produce close agreement with DNS at low Reynolds numbers, incuding the flatness factor of pair separation Malik and Vassilicos (1999).

An individual Eulerian turbulent flow field realization in KS is generated as a truncated Fourier series,

$$\mathbf{u}(\mathbf{x},t) \quad = \quad \sum_{n=1}^{N_k} \left((\mathbf{A}_n \times \hat{\mathbf{k}}_n) \cos(\mathbf{k}_n \cdot \mathbf{x} + \omega_n t) + (\mathbf{B}_n \times \hat{\mathbf{k}}_n) \sin(\mathbf{k}_n \cdot \mathbf{x} + \omega_n t) \right) \tag{7}$$

where N_k is the number of representative wavenumbers, typically hundreds for very long spectral ranges, $R_k \gg 1$. $\hat{\mathbf{k}}_n$ is a random unit vector; $\mathbf{k}_n = k_n \hat{k}_n$ and $k_n = |\mathbf{k}_n|$. The coefficients $\mathbf{A}_n$ and $\mathbf{B}_n$ are chosen such

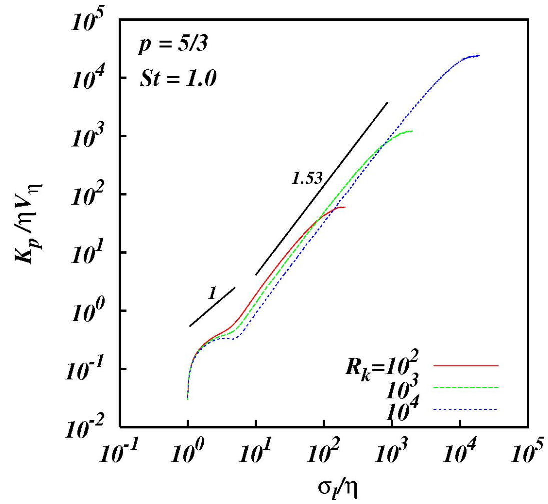

Figure 3: Same as Figure 1, except here the particle Stokes number is $St = 1.0$.

that their orientations are randomly distributed in space and uncorrelated with any other Fourier coefficient or wavenumber, and their amplitudes are determined by $\langle \mathbf{A}_n^2 \rangle = \langle \mathbf{B}_n^2 \rangle \propto E(k_n)dk_n$, where $E(k)$ is the energy spectrum in some wavenumber range $k_1 \leq k \leq k_\eta$. The angled brackets $\langle \cdot \rangle$ denotes the ensemble average over space and over many random flow fields. The associated frequencies are proportional to the eddy-turnover frequencies, $\omega_n = \lambda \sqrt{k_n^3 E(k_n)}$. There is some freedom in the choice of λ, so long as $0 \leq \lambda < 1$. The construction in equation (7) ensures that the Fourier coefficients are normal to their wavevector which automatically ensures incompressibility of each flow realization, $\nabla \cdot u = 0$. The flow field ensemble generated in this manner is statistically homogeneous, isotropic, and stationary.

The energy spectrum $E(k)$ can be chosen freely within a finite range of scales, even a piecewise continuous spectrum, or an isolated single mode are possible. To incorporate the effect of large scale sweeping of the inertial scales by the energy containing scales, the simulations are carried out in the sweeping frame of reference by setting $E(k) = 0$ in the largest scales, for $k < k_1$ Malik (2017). We choose the energy spectrum in the inertial subrange,

$$E(k) = C_k \varepsilon^{2/3} k^{-5/3}, \qquad k_1 \leq k \leq k_\eta \tag{8}$$

where C_k is a constant. The largest represented scale of turbulence is $2\pi/k_1$ and smallest is the Kolmogorov micro-scale $\eta = 2\pi/k_\eta$. A particle trajectory, $\mathbf{x}(t)$, is obtained by solving equations (1) and (2) in time. Pairs of trajectories are harvested from a large ensemble of flow realizations and pair statistics are then obtained from it for analysis.

4 Simulation Results

KS was run with the spectrum of $E(k) \sim k^{-5/3}$, for an ensmeble of about 30,000 inertial particle pairs, and the results are presented below for several inertial subranges and for different Stoke's numbers.

Figure 1 shows the pair diffusion coefficient, $K_p/\eta v_\eta$, against the rms separation, σ_l/η, when the particle Stokes number is $St = 0.25$, for different sizes of the inertial subrange as indicated. A line of slope 1 is shown for comparison with ballistic motion, and a line of slope 1.53 is shown for comparison with the fluid particle asymptotic limit.

Figures 2 to 4 are similar except for the Stokes numbers of, $St = 0.5$, 1.0, and 5.0 repectively.

The results show initial ballistic regimes, equation (5) that penetrate further and further in to the inertial subrange as R_k increases.

At long times, the inertial particle pair diffusion appears to be asymptoting towards the fluid particle non-local regime $K_p \to K_f \sim \sigma_L^{1.53}$ as $R_k \to \infty$, Malik (2017, 2018a). However, it will require bigger R_k to fully confirm.

There also exists a transition regime, over an extended range of scales, between these two limiting cases.

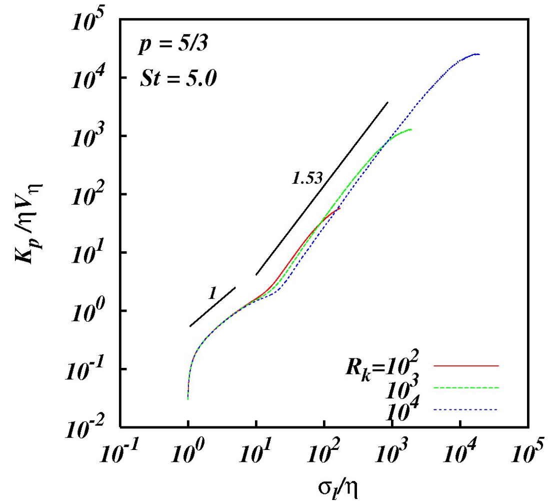

Figure 4: Same as Figure 1, except here the particle Stokes number is $St = 5.0$.

5 Discussion

A theory of inertial particle pair diffusion has been developed which extends the concept of local and non-local diffusional processes to inertial particles, Malik (2017, 2018a).

For Kolmogorov energy spectrum, $E(k) \sim k^{-5/3}$, Kinematic Simulations has been used to investigate the scaling laws for inertial particle pair diffusion in the limit of Stokes drag law. For very large inertial subranges, the long time regime approches the fluid particle non-local scaling, which vindicates our inital assumption of extending the concept of local and non-local diffusional processes to inertial particle pair diffusion.

The results indicate that inertial pair diffusion coefficient is a two parameter function $K(St,l)$ in general. For short times, the pair diffusion displays ballistic motion where the paticle inertia is dominant over the trubulence energy at that pair separation scale.

In the future we will complete the paramteric study for larger inertial subranges, and for more generalised inverse power law energy spectra, and over s wide range of Stokes numbers.

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