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Single axis volumetric µPTV for wall shear stress estimation

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Abstract

Measuring the wall shear stress (τ_W) of a flow is a challenging task, in particular at larger flow velocities due to the strong velocity gradient close to the wall. Preferably, non-intrusive measurement techniques are used to determine τ_W, since they do not disturb the flow. Thus, particle imaging techniques are a suitable tool to estimate the wall shear stress. Moreover, they provide a good combination in the accuracy of the velocity measurement as well as in the accuracy of the spatial location of the measurement. Among the particle imaging techniques, Particle Tracking Velocimetry (PTV) is more capable than Particle Image Velocimetry (PIV) for near wall measurements. Unlike the cross-correlation based approaches (as it is PIV), PTV does not suffer from spatial averaging. This results in a better spatial resolution for mean velocity fields and bias errors due to the spatial averaging over an interrogation window can be avoided. A good spatial resolution is inevitable for an accurate wall shear stress estimation, since this quantity is determined by a linear fit of the velocity profile close to the wall. Volumetric microscopic PTV offers the ability to yield accurate estimates of the wall shear stress using only a single axis for the imaging and the illumination. This technique is combined with an *in situ* calibration where calibration target needs to be placed in the measurement volume. Therefore, the technique requires an optical access with a size on the order of only 10 millimeters in diameter. Without the necessity of placing a calibration target in the measurement domain, it is still possible to conduct measurements in facilities with limited access, such as turbomachines, combustion engines, and other applications with a confined geometry. The feasibility of the technique is proven by a comparative measurement with parallax corrected PTV, shown in this article.

1 Introduction

Precise information about the wall shear stress (τ_w) is inevitable to determine the local friction force from the fluid to the solid surface, but also to access the flow state in aerodynamics or to determine the typical normalization quantities in turbulence research.

One of the major difficulties of measuring the near wall flow accurately is the fact that the viscous scales become rather small with increasing Reynolds number. In particular, at large flow speeds, it is required to measure the flow profile with micrometer resolution in wall normal direction. Particle Tracking Velocimetry (PTV) is a suitable approach for such near wall measurements for several reasons: First, PTV is a non-intrusive technique that does not disturb the flow in the measurement domain. Second, systematic errors due to spatial averaging caused by the finite size of the correlation windows like for PIV can be avoided (Kähler et al. (2012b)) and sub-pixel resolution can be achieved (Kähler et al. (2012a)). Third, unlike for laser Doppler anemometry (LDA), where the probe volume is rather large, the location of the velocity measurement can be estimated with high precision using PTV.

Among the PTV techniques, the parallax correction PTV approach has shown to yield the highest resolution of the near wall flow profile (Cierpka et al. (2013)). Even if strong vibrations are present and the thickness of the boundary layer is small, the flow profile can measured with a resolution of around 5 µm (Fuchs et al. (2018)). However, the limit of the parallax correction technique is that at larger magnifications, the particle image diameter increases and the particle images and their mirror images start to overlap, such that a correction of the wall normal location of the particle is not feasible anymore. Furthermore, optical access to the measurement domain is required at two locations, one for the camera and one for the laser, which are oriented perpendicular to each other. In some facilities such an alignment requirement can cause problems due to the limited accessibility, in particular if the facility has a more complex geometry, such as

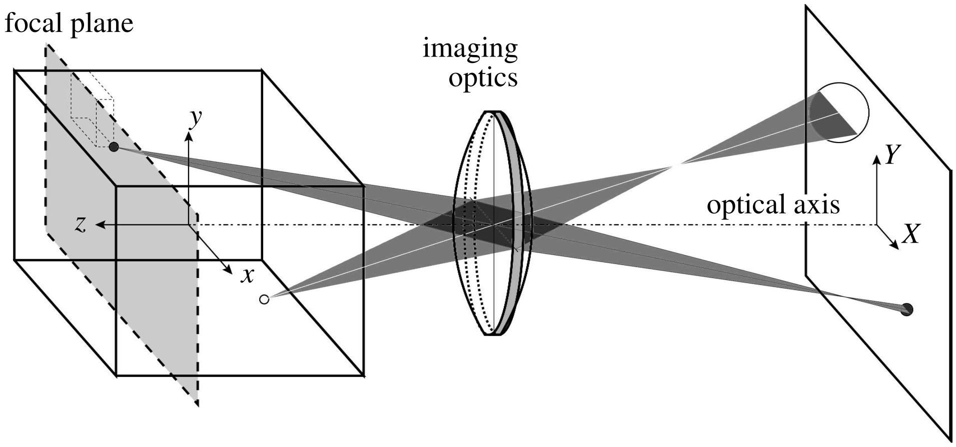


Figure 1: Defocusing imaging concept. The particle location along the optical axis is determined by the diameter of its particle image. The diameter increases with distance to the focal plane.

in turbomachinery applications.

It is the aim of the volumetric µPTV approach outlined in this paper to provide a measurement scheme that improves the particle imaging based wall shear stress estimation and that extends its applicability to larger flow velocities. The approach is based on microscopic defocusing particle imaging and the fact that the diameter changes linearly over a length that is larger than the measurement volume depth, i.e. in the direction of the optical axis.

2 Measurement principle

The µPTV measurement approach estimates τ_w by means of a linear fit of the velocity profile close to the wall. To determine the velocity profile, defocusing PTV with microscopic optics is employed. Unlike the three-aperture defocusing approach, introduced by Willert and Gharib (1992), in this defocusing technique the particle location along the optical axis is determined from the geometry of its particle image (diameter of the first diffraction ring), comparable to the astigmatism PTV method (Cierpka et al. (2010)). Figure 1 illustrates this particle location determination scheme.

The idea of this current defocusing approach for wall shear stress estimation emerged from a previous macroscopic defocusing PTV application for small measurement depths (Fuchs et al. (2016)). It is made use of the fact that the particle image diameter changes linearly over the measurement depth, which allows for an *in situ* calibration. This means that no calibration target needs to be placed within the measurement domain; a feature that is really useful if the accessibility of the domain is difficult or even impossible, like in a Tesla turbine rotor for instance (Schosser et al. (2016) and Schosser et al. (2019)). However, for the macroscopic approach the measurement domain needs to be confined by walls on both sides along the optical axis. This is not the case in this microscopic approach for wall shear stress estimation, where only one wall lies inside the measurement domain – the wall where the shear stress has to be determined.

To overcome this issue, measurements at different distances of the camera relative to the measurement domain, yielding multiple velocity profiles, as shown in Fig. 2. For each of these profiles, a linear fit of the near-wall velocity values results in a particle image diameter where the displacement is zero. From these diameter values along with the known measurement locations, i.e. the different camera distances to the wall, a function relating the particle image diameter and the distance from the camera can be determined, as shown in Fig. 3. As a result, the particle location relative to the wall can be calculated. An assumption that has to be made is that the diameter of the particle image changes linearly with distance to the camera, at least in the z range of the measurement domain. As shown in Fuchs et al. (2016), this calibration procedure can or even needs to be done depending on the sensor location of the particle image, since the geometry of the particle images can be influenced strongly by optical aberrations, such as spherical aberration and field curvature, which are most important in microscopic imaging.

However, the necessity of an *in situ* calibration comes along with inaccessible measurement volumes. If it is possible to place a target in the measurement domain, then the camera does not have to be moved for

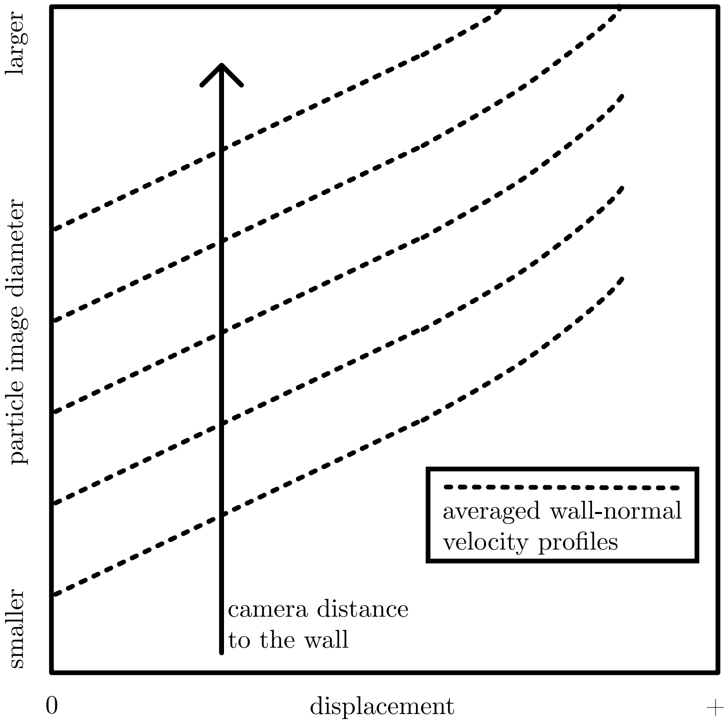


Figure 2: Principle of the *in situ* volumetric µPTV wall shear stress estimation approach: The particles are tracked at different camera positions relative to the measurement domain, yielding a linear scaling of the particle image diameter over the wall distance of the particle image. Using this linear relation the particle distance from the wall can be calculated.

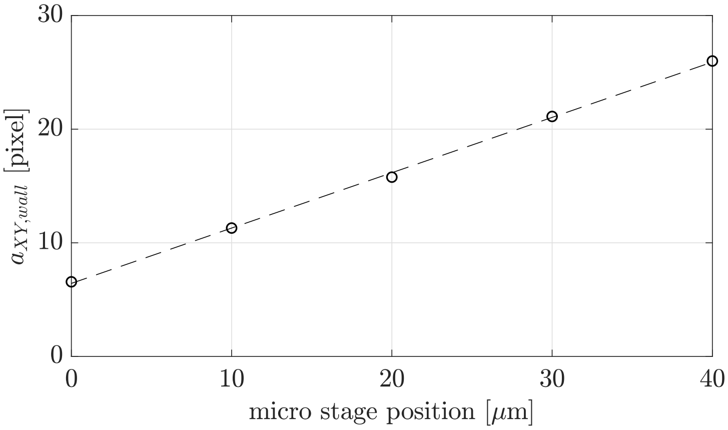


Figure 3: An experimentally determined calibration function, relating the particle location along the optical axis to the particle image diameter. In this case, the scaling is 2.05 µm/pixel in wall normal direction. The measurements were conducted at 5 different locations with a step width of 10 µm.

the measurements. Such a calibration procedure, using a pinhole matrix, is described in Fuchs et al. (2014), and is equal to the macroscopic APTV calibration procedure.

3 Measurement set-up

The intriguing feature of this approach is its straightforward set-up. It requires standard microscopic particle imaging equipment, i.e. a camera, a double-pulse Nd:YAG laser, and a microscope objective. In addition, if an *in situ* calibration is required, the camera and the objective need to be mounted on a micro-translation stage. Moreover, the technique requires only a single optical access for both the camera and the laser. Thus, a window width and height on the order 10 mm is sufficient. A principle sketch of the set-up is shown in Fig. 4.

For the illumination of the volume, the direct light beam of the laser can be used without any exception. In this configuration it is not feasible to restrict the illumination to the measurement depth, which is around 100-200 µm, depending on the microscope objective. Along the optical axis, i.e. in z direction, the measurement volume is simply limited by the particle image diameter. The diameter quickly increases away from the wall, and therefore also the intensity of the particle image, only appearing as background noise. Altogether, the illumination arrangement is very straightforward, since a precise alignment is not necessary.

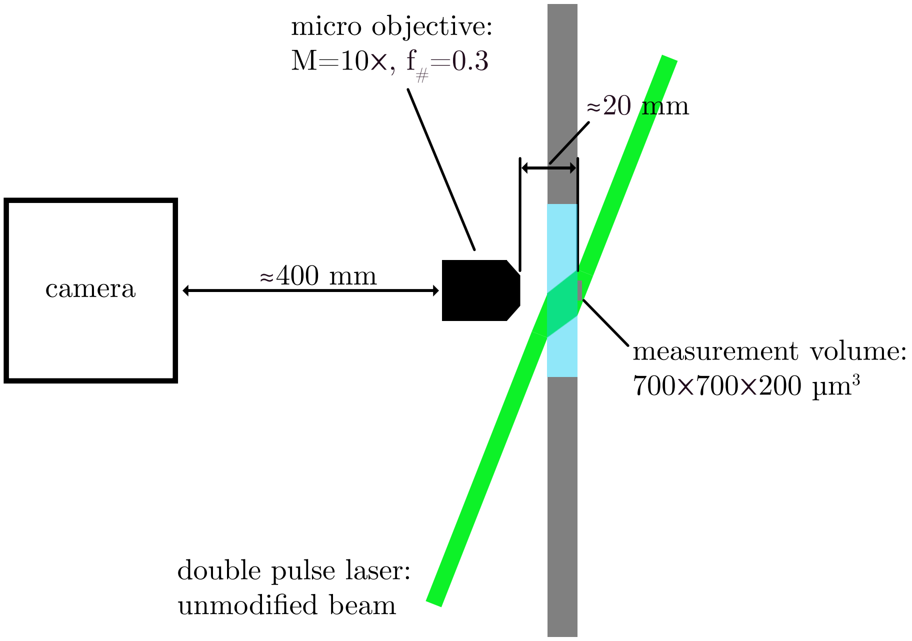


Figure 4: Imaging set-up: The laser and the objective share a single window. A window width and height on the order of 10 mm is sufficient for this volumetric µPTV approach.

Compared to the macroscopic approach outlined in Fuchs et al. (2016), the microscopic approach yields a much higher sensitivity in regards of the diameter change of the particle image of a particle along the optical axis. In the case of the microscopic objective *Zeiss* 420330-9901, having a numerical aperture of $f_{\#} = 0.16$ and a magnification of $M = 5\times$ according to the manufacturer, the diameter change over z (wall normal direction), yields 2.05 µm/pixel. Compared to the macroscopic defocusing approach outlined in Fuchs et al. (2016), with a change of the particle image diameter of around 100 µm/pixel along the measurement depth, this is a significant resolution improvement. However, it has to be considered that the determination of the diameter of the particle image is more error prone due to vibrations and also due to the less significant signal of the edge of the particle image. Therefore, an accuracy of the diameter determination of 0.1 pixel as in the case of the macroscopic approach cannot be reached with the current particle image geometry determination method. Correction methods, using microscopic contaminations of the optical access window, can improve the location uncertainty arising from vibrations.

When imaging at large magnifications, as it is the case for microscopic imaging, the assumption of a purely diffraction limited imaging is not valid anymore. Instead, the physical size of the tracer particles has an influence on the particle image diameter. To analyze this influence, Olsen and Adrian (2000) established an equation to determine the particle image diameter for microscopic imaging systems:

$$d_{PI}(z) = \left(M^2 d_\mathrm{p}^2 + 5.95\,(M+1)^2 \lambda^2 f_\#^2 + \frac{M^2 z^2 D_a^2}{(s_0+z)^2} \right)^{1/2}, \tag{1}$$

where M is the magnification, d_p is the seeding particle diameter, λ is the wave length of the illumination light, D_a is the aperture diameter of the objective, and s_0 is the distance between objective and the measurement domain. Figure 5 shows the particle image diameter relative to the focal plane for several particle sizes, namely 1, 2, 5, and 10 µ, and a magnification of M = 13. It is evident that further away from the focal plane, in the defocused region where the measurement volume of this µPTV is situated, the influence of the particle size becomes much smaller, almost negligible. Since for the current approach either DEHS or water-glycol seeding particles with a size distribution around 1 micron are used, it is still a valid assumption to say that the particle image diameter solely depends on the location of the particle along the optical axis and not on its size.

For the feasibility study outlined in this text, the magnification of the imaging set-up, using a *PCO sCMOS* camera and a *Zeiss* 420330-9901 micro objective, yields around M = 13. However, it is also possible to use an M = 10× objective, roughly doubling the sensitivity with a stronger diameter change of the particle along the optical axis. Such an optical configuration has a magnification of around 25 using a *sCMOS* camera.

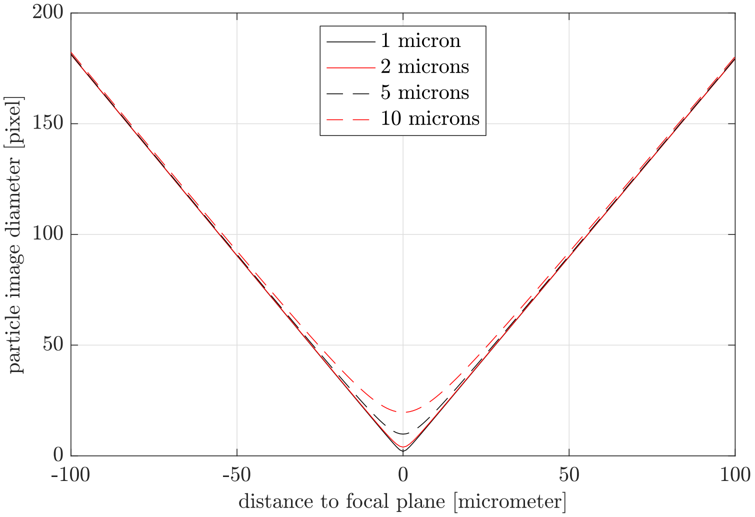


Figure 5: The particle image diameter as a function of the distance to the focal plane for different tracer particle sizes (1, 2, 5, and 10 µm), according to Olsen and Adrian (2000). Close to the focal plane the tracer particle size has a significant influence on the particle image diameter. Further away from the focal plane, the influence of the tracer particle size becomes smaller and smaller. It is negligible for this µPTV approach, where DEHS or water-glycol tracers can be used which have a size distribution of around 1 µm.

4 Feasibility – comparison with parallax corrected PTV

To assess the feasibility of the τ_w estimation using volumetric µPTV, a comparison measurement with parallax corrected PTV was carried out. The set-up for this experiment is shown in Fig. 6, where the flow over a short glass plate was measured. The flow is provided by an open jet wind tunnel and it has to be mentioned that the purpose of this experiment is solely to compare the two measurement approaches. Of course, the parallax approach serves as a reference, since this measurement technique has shown to provide reliable near-wall velocity profiles.

Figure 7 shows the velocity scatter data of the two approaches side by side; left is the parallax corrected approach and on the right sight is the μPTV approach for a single measurement location. It becomes apparent that the measurement domain dimension in wall-normal direction is much smaller for the μPTV approach with 100-200 μm, whereas it is larger than 2 mm for the parallax approach which is an order of magnitude larger.

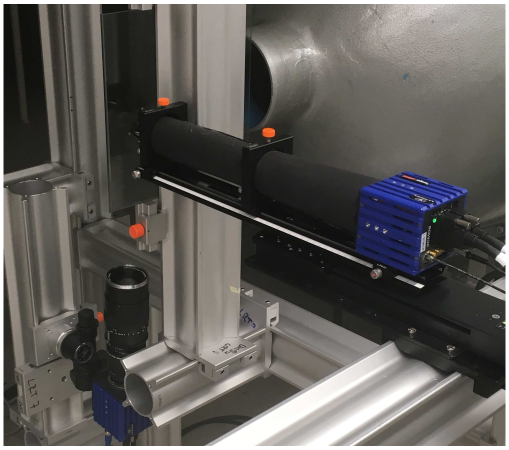
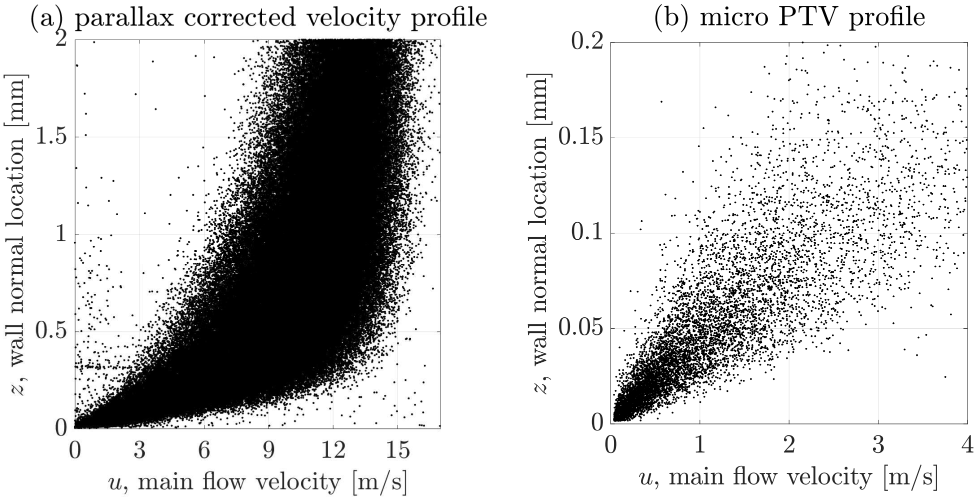


Figure 6: Set-up for the comparative measurements for wall shear stress estimation. The lower camera is the parallax corrected PTV set-up. Horizontally installed on a micro-translation stage, the volumetric μPTV camera.



(a) parallax corrected velocity profile

(b) micro PTV profile

Figure 7: Comparison of the velocities measured close to the wall for parallax corrected PTV, serving as a reference, and volumetric μPTV. A good agreement between the estimated τ_{w} values was achieved: $\tau_{\mathrm{w,\,parallax}} = 0.461\ \mathrm{N/m^2}$ and $\tau_{\mathrm{w,micro}} = 0.458\ \mathrm{N/m^2}$.

Calculating the wall shear stress from the two measurement approaches a good agreement is reached. The parallax approach yields $\tau_{\mathrm{w,\,parallax}} = 0.461\ \mathrm{N/m^2}$, whereas μPTV yields $\tau_{\mathrm{w,micro}} = 0.458\ \mathrm{N/m^2}$, which is an average value of the measurements at different camera distances relative to the measurement domain. The measurements were conducted subsequently and not a the same time.

To derive the calibration function, μPTV measurements at different distances to measurement volume were conducted, specifically 5 positions in steps of 10 µm. There is only a small fluctuation of the wall shear stress estimates for these measurements yielding $\tau_{\mathrm{w,micro}} = \overline{0.458} \pm 0.01$ N/m^2, proving the reproducibility of the measurements.

It is worth mentioning that the pulse separation was 7 µs for these measurements. The *sCMOS* cameras used in these measurements can go down to a frame separation of only 0.25 µs, which leaves enough room for measurements at larger velocities.

5 Conclusion

Volumetric µPTV based on defocusing imaging is a robust and reliable tool to measure the wall shear stress. It has shown to yield accurate τ_{w} estimates using an *in situ* calibration. The technique requires only a small optical access, since the camera and the laser can share the same access window. Without the necessity of larger constructional modifications, the technique opens up new applications for the τ_{w} determination, in particular for smaller, more confined and complex geometries, such as turbomachinery facilities, combustion engines, among others.

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