Acta Polytechnica CTU Proceedings **36**:76–83, 2022
https://doi.org/10.14311/APP.2022.36.0076

Published by the Czech Technical University in Prague

Uncertainty Assessment for the Bayesian Updating Process of Concrete Strength Properties

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Abstract. Reassessment of infrastructure buildings has become an essential approach to deal with increasing traffic loads on ageing infrastructure buildings and to verify the service-life of those structures. Good estimation of the actual material properties is highly relevant for reliable structural reassessment. Although this holds for all building materials, the importance of good parameter estimation is of special importance for concrete structures, where the strength properties show relatively high variation and where the nominal strength properties tend to be too conservative. Modern design guidelines allow to make use of scientific methods such as Bayesian Updating of material properties to enable a more realistic consideration of the actual material properties in the reassessment of existing structures. However, guidelines for application and experience with those methods are not yet reported much or are rather vague [1]. The presented study focuses on the effect of the Bayesian Updating process for material parameters with special emphasis on the number and sampling location of test specimens as well as on the accuracy and confidence in the obtained posterior distribution, since sampling also includes a certain margin of uncertainty. The investigation on the methodological potential and on the uncertainty margin in the updating process in this contribution uses a batch of 14 test results on the concrete compressive strength obtained from drill cores along with the inherent measurement uncertainties from the testing procedure. After a short review of Bayes' Theorem, the Markov Chain Monte Carlo Method (MCMC) and the bootstrap methodology, all combinations of subsamples of size 1, 3 and 5 specimens were built and provided to the Bayes' updating procedure via MCMC to determine the posterior distributions. The series of obtained posterior distributions for a certain subsample was used to determine the uncertainty in the Bayesian Updating process by evaluation of the scatter in the expected value, the standard deviation and the 5 %-quantile of the updated distribution. The simulations show the importance of an adequate sample size and quantify the uncertainties arising from the limited number of observations.

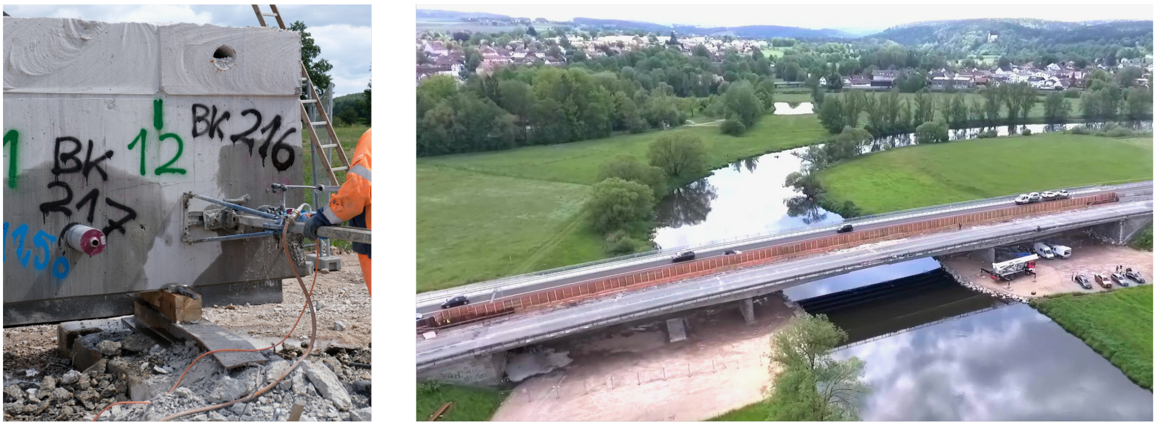
Keywords: Bayesian updating, bootstrapping, burn-in, concrete compressive strength, Markov chain Monte Carlo, MCMC, Metropolis algorithm, Roding Bridge, structural reassessment.

1. Introduction

Continuously increasing life-loads and ageing of infrastructure are the most important reasons for the requirement of reassessment of existing infrastructure buildings. For this purpose, the correct estimation of the actual material properties is of high practical relevance. This is considered specifically difficult for concrete structures, since the material strength depends on various factors and as consequence of this shows considerable scatter. In many cases the nominal values in the design codes turn out to be rather conservative, since they must represent the expected strength values with a certain safety margin. Material tests on specimens taken from the structure are worth a lot to verify the expected strength class, but can't be used directly for the structural reassessment process. As a consequence of this, the scientific motivation strives for a more advanced methodology and consideration of the measurement results for structural reassessment. Statistical methods such as Bayes' Theorem play an increasingly important role for interpretation and exploitation of data and setup of specifically tailored reassessment of individual structures.

More precisely, Bayes' Theorem allows to update prior information from literature or other sources by direct or indirect observations. However, spatial variations of material properties within the structure and uncertainties in the measurement process lead to follow-up questions for the application of such methods, especially when the sample sizes are small.

In this paper, the methodology of stochastic updating is shown on the example of the compressive strength of concrete using Bayes' Theorem via Monte Carlo Approximation. Similar studies have been discussed in e.g. [2] using theoretical data sets. For this



(a) (b)

Figure 1. (a) Sampling of the drilling cores and (b) bird's eye view of the investigated bridge in Roding.

purpose, a limited number of destructive material test results from a large-scale structure are used to show the outcome of the updating procedure for different sizes of the data set. For the Bayesian updating process, all possible subsamples of size 1, 3 and 5 core samples are composed from the entire data set of 14 measurements. For all those subsamples the posterior stochastic models are calculated. The resulting expectation, the standard deviation and the 5 %-quantile of the subsamples are used to derive a distribution that shows the uncertainties related to the choice of the measurement locations and cope with the aspect of spatial variation. The study is conducted using Markov Chains which have proven to be an excellent tool for the implementation of Bayesian Updating in the framework of a Monte Carlo simulation. As a sampling strategy, the Metropolis Algorithm has been implemented in MATLAB to conduct the more than 2,300 simulations.

2. Methodology

2.1. Evaluation of Compressive Strength

Within a research project dealing with the updating of finite-element-models for a refined calculation of existing structures using different kinds of experimental data, 14 core samples were taken at different locations during the demolition of a specific bridge located in Roding, Bavaria. The three-span box girder bridge dating from 1965 was made up from site concrete of strength class B300 corresponding to the strength class C20/25 in current design standards in Europe [3]. For the determination of the sampling locations, a minimum distance of 3.5 m between those locations corresponding to the requirements on the correlation length of compressive strength properties described in [4] was respected. Figure 1(a) gives an impression of the drilling process on a bridge part after demolition while Figure 1(b) shows a bird's eye view of the structure. For further information concerning the research project the data set originates from, please refer to [5–7].

The material testing of the cylinder strength of hardened concrete specimens was conducted according to DIN EN 12390-3:2019-10 [8] using cylindrical drill cores with 150 mm in diameter and about 300 mm in height. The results of the material tests are given in Table 1. The material tests were carried out on a compression test machine of type MTS (max. force 5 MN). The accuracy of the strength testing procedure can mainly be attributed to the measurement of the pressure area with an estimated homoscedastic measurement uncertainty of $\sigma = 0.05\,\mathrm{N/mm^2}$ [9].

Specimen No.	Test Result $\left[\mathrm{N/mm^2}\right]$
No. 1	59.6
No. 2	55.1
No. 3	45.7
No. 4	48.2
No. 5	58.0
No. 6	58.0
No. 7	70.4
No. 8	59.9
No. 9	73.4
No. 10	38.0
No. 11	67.9
No. 12	63.6
No. 13	57.2
No. 14	73.0

Table 1. Data from compressive strength testing.

Due to additional sources of uncertainty from the reading of the results and the handling of the specimen, the uncertainty in the data shall be assumed as a normal distribution with an estimated standard deviation of $\approx 1\,\mathrm{N/mm^2}$.

In addition to the cylinder strength testing, the Young's modulus was determined. Since this contribution focuses on the methodological approach, the test results are not reported here.

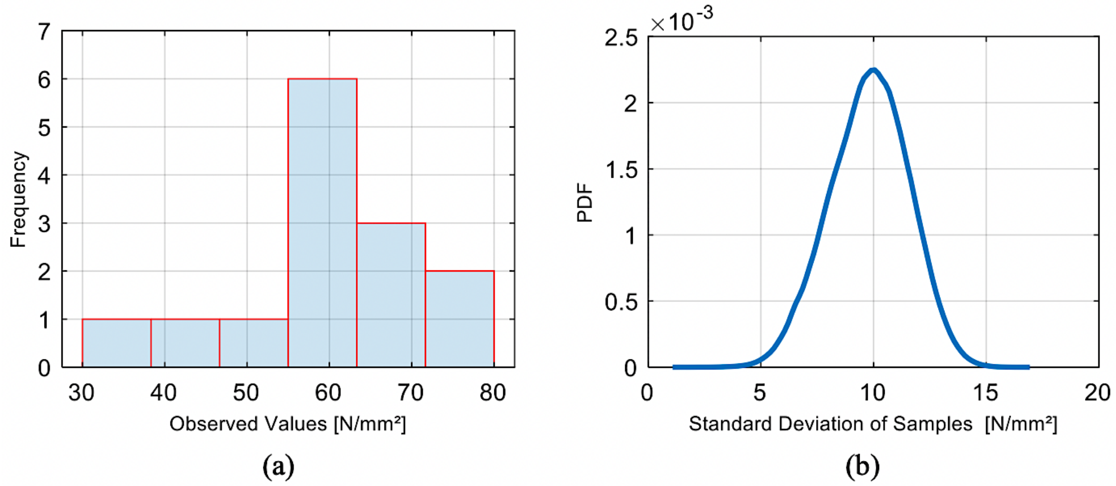


FIGURE 2. (a) Histogram of the observed concrete strength values (b) Kernel density estimation of the standard deviation of the bootstrap samples as statistical scatter estimator.

2.2. STATISTICAL ANALYSIS OF THE ORIGINAL DATA SET

The data set from section 2.1 has been examined by means of descriptive statistics. Figure 2(a) shows a histogram of the data set using 6 bins. The mean value of the 14 samplesis 59.1 N/mm^2, while the standard deviation in the data set is 10.3 N/mm^2.

In order to obtain a good estimation on the point estimators of the population from one specific and limited set of test results, the bootstrapping methodology can be used. In this methodology, the available data set is resampled with replacement to generate additional vectors of resampled data that can be compared among each other. The distribution of a certain estimator, in this case the expected value, the standard deviation and the 5 %-quantile of the compressive strength, is subsequently evaluated using the vectors of resampled data. The derived statistics can thus be seen as an evaluation of the uncertainty for the point estimators.

The standard deviation of the point estimator for the measurement results can be computed to be ≈ 2.7 N/mm^2 (expectation) and ≈ 1.8 N/mm^2 (standard deviation), while it is about 4.6 N/mm^2 for the 5 %-quantile. This indicates that the gained data give a sufficient approximation, but still leaves space for further research which uses more specimens. Further details on the bootstrap methodology can be found, e.g. in [10–12].

In order to visualize the assumed PDF of the bootstrap-samples, kernel density estimation can be employed to smoothen the graph. For the kernel density shown in Figure 2(b), a number of 10^6 resamples have been evaluated.

2.3. BAYESIAN FRAMEWORK

The application of Bayes' Theorem allows for the updating of information based on knowledge from observations i.e. measurements. As a basis for updating, the distribution $p(f_c)$ has to be established which states our belief in the occurrence of a certain compressive strength.

In order to formalize our belief in the appearance of a certain observation under uncertainty, the conditional probability $p\left(\tilde{D}_{f_c}|f_c\right)$ has to be determined which represents the probability to observe a cylindrical compressive strength $\tilde{D}_{f_c}$ conditional on the value of f_c. This expression is also called likelihood-function and is abbreviated here $L\left(f_c|\tilde{D}_{f_c}\right)$. The formal expression for $L\left(f_c|\tilde{D}_{f_c}\right)$ for a number of n observations and a measurement uncertainty of σ is given in Equation 1.

$$L\left(f_c|\tilde{D}_{f_c}\right) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{f_{ci} - \tilde{D}_{f_{ci}}}{2\sigma^2}\right) \quad (1)$$

As input for the simulations, the test data from Table 1 has been used and is denominated $\tilde{D}_{f_c}$ in Equation 1. The measurement uncertainty inherent to observations using imperfect measurement devices finds its way into the formula using the standard deviation, which has been determined in section 2.1 to be 1 N/mm^2 . For the specification of the applied probability function for $p(f_c)$, please refer to chapter 2.5.

Using these basic relations, Bayes' Theorem can be stated when normalized by the probability $p(\tilde{D}_{f_c})$. The fundamental equation of Bayes' Theorem is given in Equation 2. For a more detailed review on Bayesian techniques please refer to [13–15].

$$p\left(f_c|\tilde{D}_{f_c}\right) = \frac{p(f_c)p(\tilde{D}_{f_c}|f_c)}{p(\tilde{D}_{f_c})} = \frac{p(f_c)L(f_c|\tilde{D}_{f_c})}{p(\tilde{D}_{f_c})} \quad (2)$$

2.4. MARKOV CHAIN MONTE CARLO

As the evaluation of the posterior distribution is in many cases not feasible in an analytical way, Monte Carlo techniques are to be used for the computation

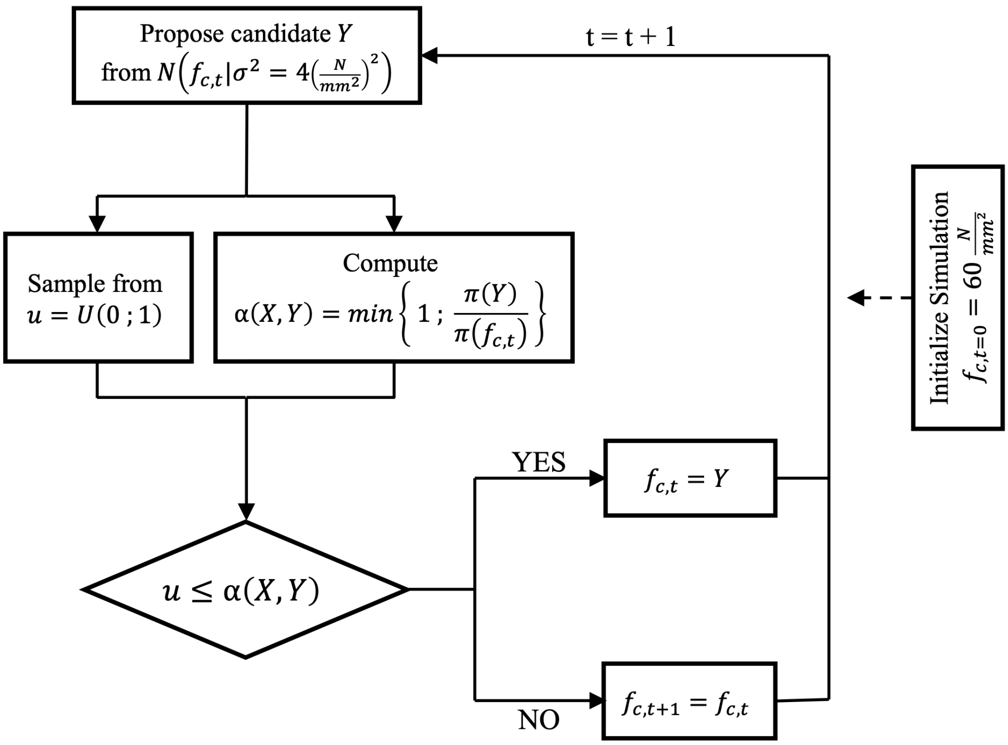


FIGURE 3. Flowchart of the applied algorithm for sampling from the posterior distribution $\pi(\cdot)$.

of posteriors. The basis of the Markov Chain Monte Carlo (MCMC) methodology is the construction of one or more Markov Chains. In a discrete Markov Chain the sampled values of the variable only depend on the preceding sample, but not on the states sampled before.

The Markov Chain explores the posterior which is its stationary distribution. However, in order to ensure this property, a certain burn-in sequence of samples has to be cut off from the sample set [16].

In this study, a single-chain algorithm has been implemented in MATLAB using Metropolis Sampling. As a proposal distribution, a normal function $X \sim N\left(0, \sigma^2 = 4\,(\mathrm{N/mm^2})^2\right)$ has been used and a burn-in of 200 samples has been implemented giving reasonable rates of acceptance. As a starting value $60.0\,\mathrm{N/mm^2}$ was applied. The entire length of the Markov Chain has been determined to be 20,200, which leads to a number of 20,000 samples for the construction of the posterior. The applicability of these parameters is proven in chapter 3.1.

A flowchart of the applied algorithm is given in Figure 3, where the posterior distribution is abbreviated by $\pi(\cdot)$. Further information for the implementation and a good overview on the subject is given in [17].

2.5. Choice of Prior Distribution

The prior information on the strength property is a basis for the updating procedure and thus has to be determined properly. Therefore, a literature review has been conducted in order to determine the type of distribution, the expectation and the standard deviation.

The test specimens were drawn from a structure that was built in concrete. According to the structural analysis documentation of the structure, concrete of material strength class B300 according to the former design code DIN 1045:1959-11 [18] was used in the sampled parts.

This former designation for a concrete strength class corresponds to class C20/25 in DIN EN 1992-1-1 [19] as described in [3], resulting in an expected strength value of $f_{cm} = 28\,\mathrm{N/mm^2}$. According to [20], a standard deviation of $\sigma \approx 5N/mm^2$ shall be assumed. The probability distribution function is approximated with a normal distribution [21]. Consequently, the prior can formally be expressed by $X \sim N\left(28\,\mathrm{N/mm^2}, \sigma^2 = 25\,(\mathrm{N/mm^2})^2\right)$.

For further insight into the probabilistic modelling of historic concrete structures and the derivation of a truncated distribution function on the basis of historic quality regulations, please refer to [22].

2.6. Workflow of the Conducted Procedure

In order to study the methodological effect, i.e. the potential of the methodology as a measure of how much the characteristic quantile in the updated "posterior" distribution is increased in comparison to the nominal strength value, subsets of different size were built from the original data set with 14 test results. The sizes of those subsamples were chosen at random for studying purposes and contained 1, 3 or 5 test specimens respectively. Since all possible combina-

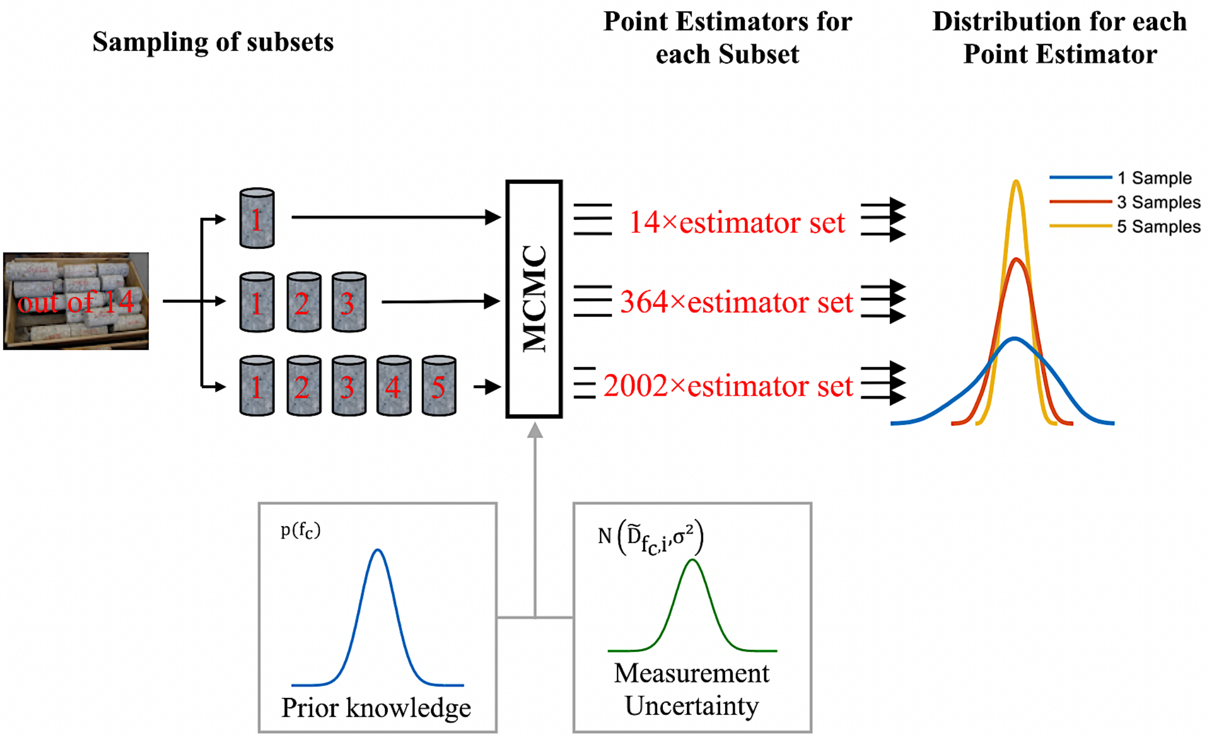


Figure 4. Illustration of the procedure.

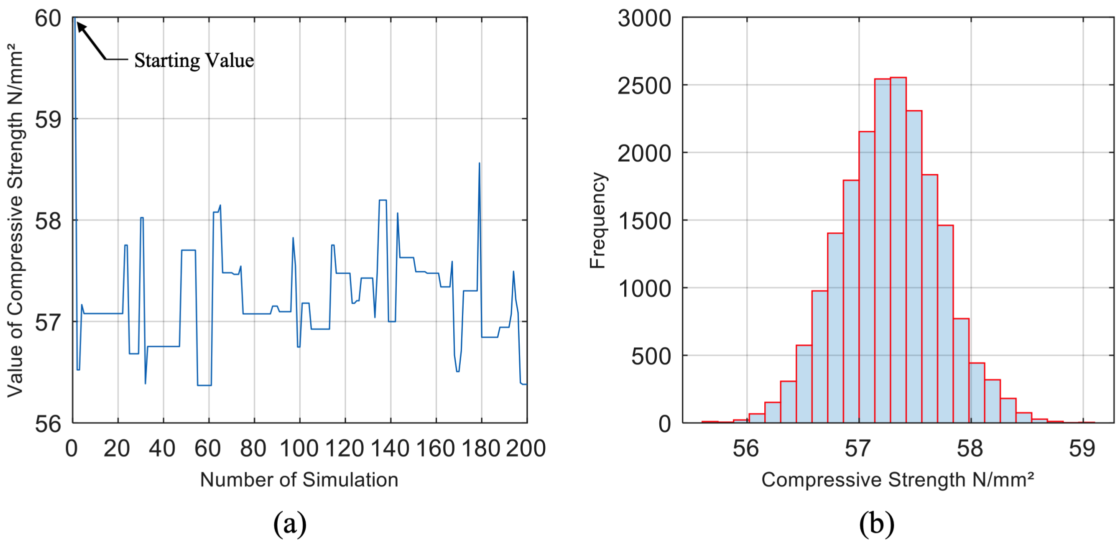


Figure 5. (a) Traceplot of the first 200 samples for a simulation using 5 measurements (No. 1,3,4,7,12); (b) Histogram of the samples from the posterior distribution using 5 measurements (No. 1,3,4,7,12).

tions of test results for the mentioned subsample sizes were built, the uncertainty in the posterior distribution resulting from the sampling pattern is addressed simultaneously.

This results in 14 subsamples of size 1, whereas 364 subsamples of size 3 and 2002 subsamples of size 5 were created and analysed.

For all subsamples of a certain size, the corresponding posterior distributions are stored and empirical probability density functions for the different point estimators are calculated after completion of the updating process for all subsets.

The computed mean values, standard deviations and the 5 %-quantiles of the simulations are presented as a kernel density-PDF and empirical CDF in section 3.

The workflow is illustrated in Figure 4.

3. Results and Discussion

3.1. Determination of Burn-in and Convergence

To ensure the validity of the computed distributions, a small study is conducted that verifies the burn-in and the number of MC-samples taken for the exploration

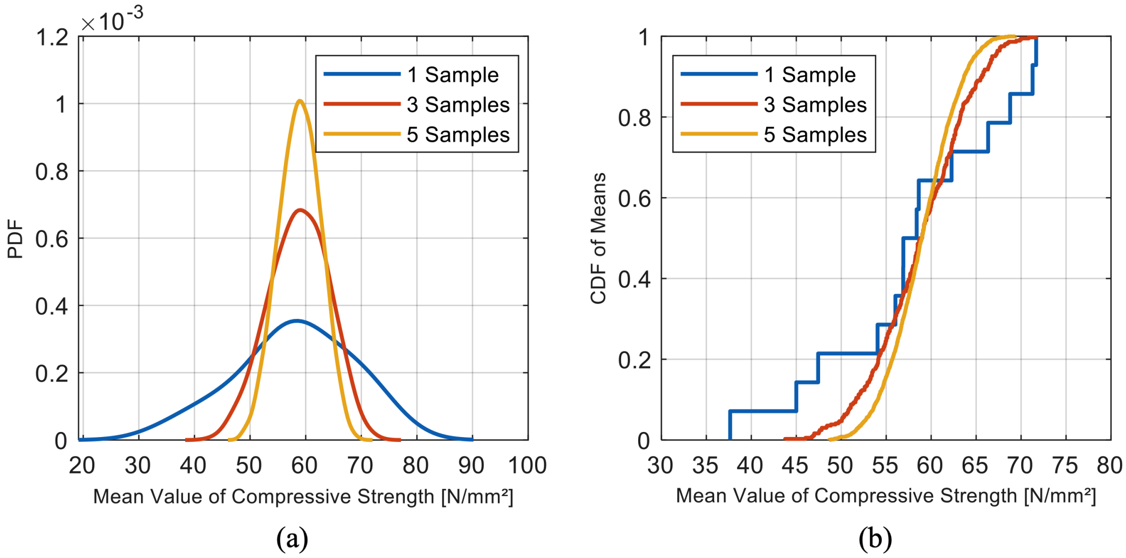


Figure 6. (a) Kernel density function and (b) eCDF of the simulation results of the mean value using 1, 3 and 5 drill cores.

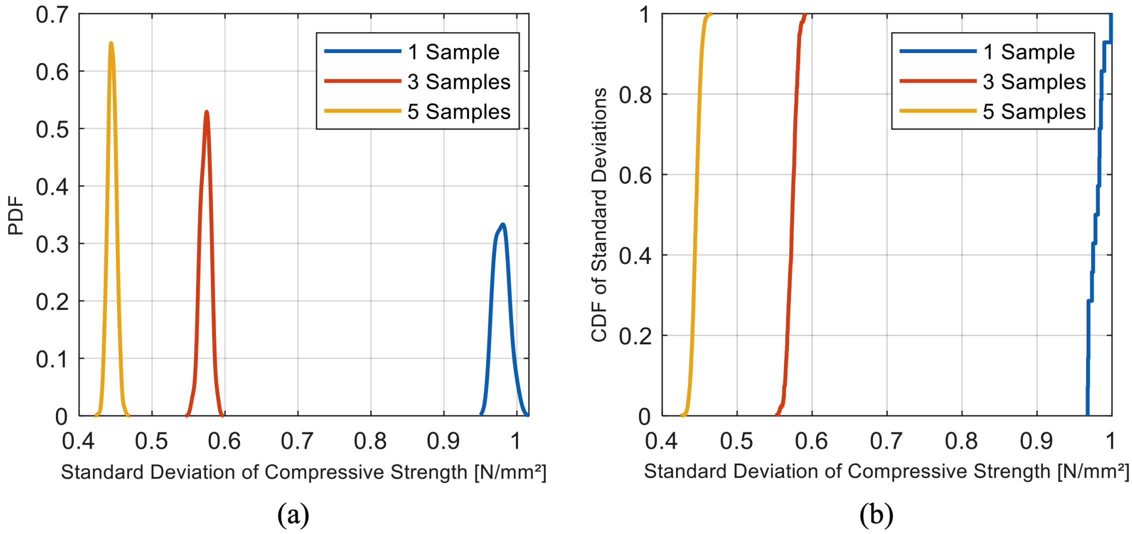


Figure 7. (a) Kernel density function and (b) eCDF of the simulation results of the standard deviation using 1, 3 and 5 drill cores.

of the posterior. Figure 5(a) shows a representative traceplot which indicates that a burn-in-sequence of 200 is more than sufficient to make the samples independent from their starting value.

The convergence of the results has been determined differently to the recommendations in [17] via bootstrapping (see section 2.2) using 10^4 replications. The maximum of the length of the 95 %- Bootstrap Confidence intervals for the applied number of 20,000 samples (without consideration of burn-in) can be determined to be at maximum $0.03\,(\mathrm{N/mm^2})$ for the mean values, $0.03\,(\mathrm{N/mm^2})$ for the standard deviation and $0.02\,(\mathrm{N/mm^2})$ for the simulations of 5 %-quantiles. The uncertainty in the recording of this posterior distribution thus can be seen as sufficiently small to ensure validity on the results from section 3. A histogram of one of the chains for 20,000 samples is given in Figure 5(b).

3.2. Uncertainty in the Simulated Mean Values

The uncertainty that can be attributed to the determination of the posterior distributions shows significant differences for different quantities of applied observations.

When only one sample is used, the standard deviation of the mean values is at $\sigma \approx 10\,(\mathrm{N/mm^2})$, while a number of 3 or 5 core samples shows variations of $\sigma \approx 5\,(\mathrm{N/mm^2})$ and $\sigma \approx 3.7\,(\mathrm{N/mm^2})$. Figure 6(a) and Figure 6(b) show a PDF and the empirical CDF of the data. The jumps visible in Figure 6(b) stem from the limited number of drawings.

The simulation results indicate that from a certain sample size, the accuracy of a simulated expected

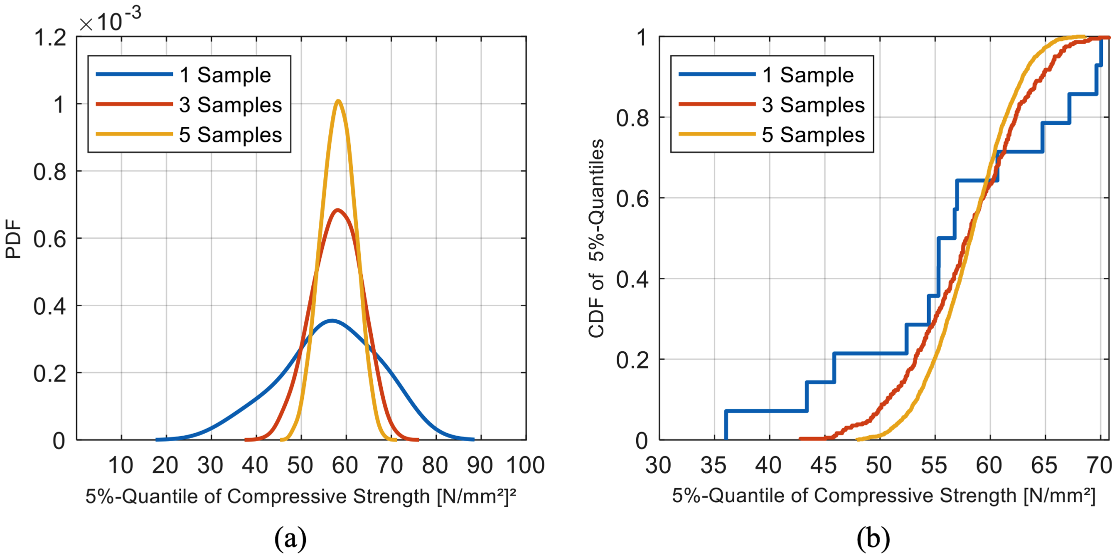


Figure 8. (a) kernel density function and (b) eCDF of the simulation results of 5 %-quantiles using 1, 3 and 5 drill cores.

value grows subproportionally and reaches reasonable results for a number of 3 core samples.

3.3. Uncertainty in the Simulated Standard Deviation

Similar to the study from 3.2, also the empirical standard deviations of the computed MCMC-samples have been computed. The peak value of the results using only one core sample for updating is at about 1 (N/mm^2), which is roughly the assumed measurement uncertainty. For samples of 3 and 5 cores, the simulated values of standard deviation are smaller and show less scatter. Figure 7 gives both the kernel density (a) and the empirical CDF (b) for the simulation results of standard deviations across the different densities.

3.4. Uncertainty in the Simulated 5 %-Quantiles

The 5 %-quantile is of high interest for practical engineering problems and is defined as the applicable characteristic value in most standards. Again, the deviation of the 5 %-quantile is significantly higher for a sample set of size one and reaches more stable values for samples of 3 and 5. The respective values are $\sigma \approx 10,\ 5.2$ and 3.7 (N/mm^2). Besides the absolute values and the distribution functions given in Figures 8(a) and 8(b), the results also reveal the enormous potential of updating information on concrete compressive strength for the practical application in the reassessment of infrastructure buildings. In this example, the characteristic value of a B300 ≈ C20/25 can roughly be estimated to be at least 35 (N/mm^2) instead of 25 (N/mm^2).

4. Conclusions

This contribution reports about application of a Bayesian Updating framework in civil engineering. More precisely, the compressive strength of concrete is subject to this procedure in order to study the potential and the characteristics of such an algorithm. The studies are carried out using a data set taken from a real large-scale concrete bridge and the aspects of spatial variation as well as uncertainties in the sampling of the compressive strength from the bridge are taken into account explicitly. In order to address those aspects, the analysis was carried out for different subsamples. The framework shown here applies Bootstrapping for first statistical evaluation of the available data set. Subsequently, the Markov Chain Monte Carlo Method was chosen and implemented to carry out Bayesian Updating.

By conducting a great number of MCMC simulations for all possible combinations of single measurement values according to the subsample sizes, uncertainties from measurement were included appropriately in the analysis. The results obtained for the example in this contribution revealed considerable scatter in the point estimators of the posterior distributions, especially for the mean value and the 5 % quantile. As expected, the scatter in those parameters was significantly higher for the rather academic subsample of size one. For the subsamples of size 3 and 5 a certain trend of convergence was observed. A minimum of three samples is thus recommended. The effect of the presented methodology is to be measured best by the 5 % quantile of the concrete compressive strength. By comparing the nominal compressive strength of a C20/25 according to the design guidelines with the 5 % quantile after application of the sampling and updating procedure, the algorithm proposes an increase from 25 to about 35 N/mm^2 corresponding to nearly

50 % increase in the concrete compressive strength. However, the simulations unveiled considerable scatter margins on the point estimators. With regard to the 5 % quantile of the concrete compressive strength, the uncertainty in the point estimators resulted in an uncertainty range between 16 to 5 % depending on the size of the subsample. The posterior distribution is thus not a unique stochastic model, since the sample values and the sampling process include some uncertainties. The presented paper contributes to uncertainty quantification in Bayesian Updating processes in consideration of spatial variability of the drawn samples and measurement uncertainty.

Acknowledgements

The author's team would like to thank company Max Bögl, Sengenthal, Germany, for providing and operating the drilling equipment.

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